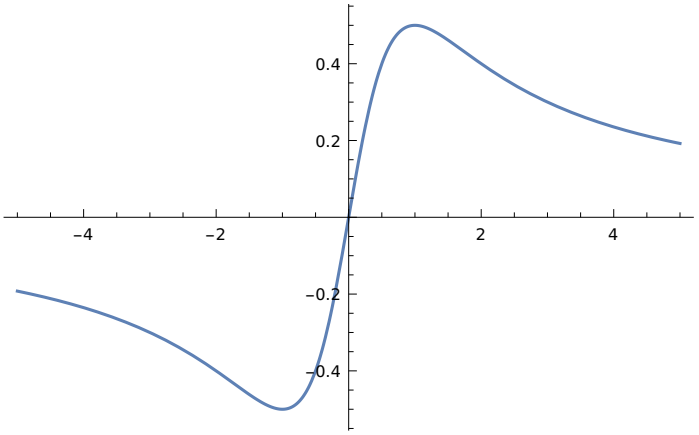
Chapter 12:
Getting started with Mathematica

Q 1. Graph each of the following functions.

In[28]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[29]:=
```
Plot[f[x], {x, -5, 5}]
```

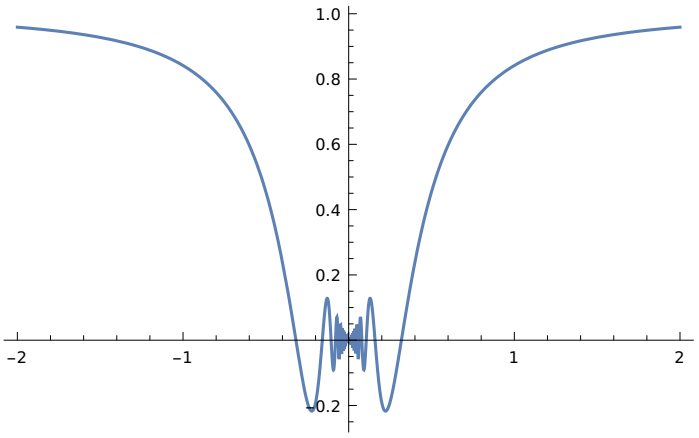
Out[29]=



In[30]:=
```
g[x_] := x Sin[1 / x]
```

In[32]:=
```
Plot[g[x], {x, -2, 2}]
```

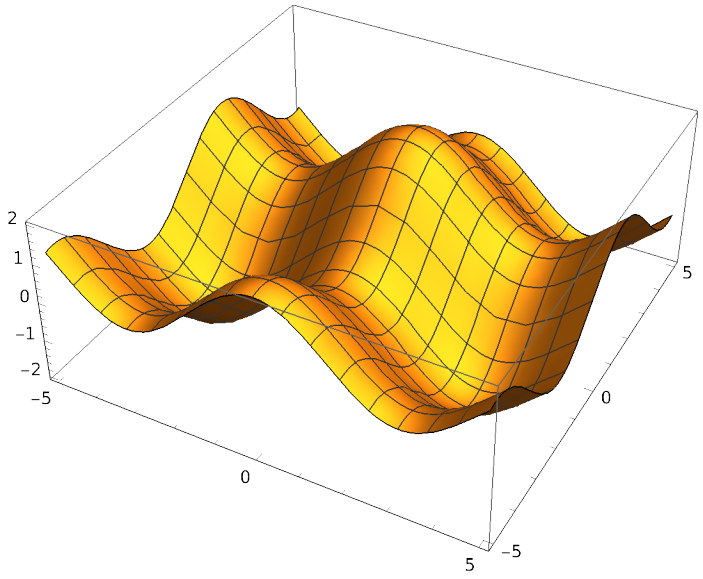
Out[32]=



In[33]:=
```
h[x_, y_] := Cos[x] + Sin[y]
```

In[34]:=
```
Plot3D[h[x, y], {x, -5, 5}, {y, -5, 5}]
```

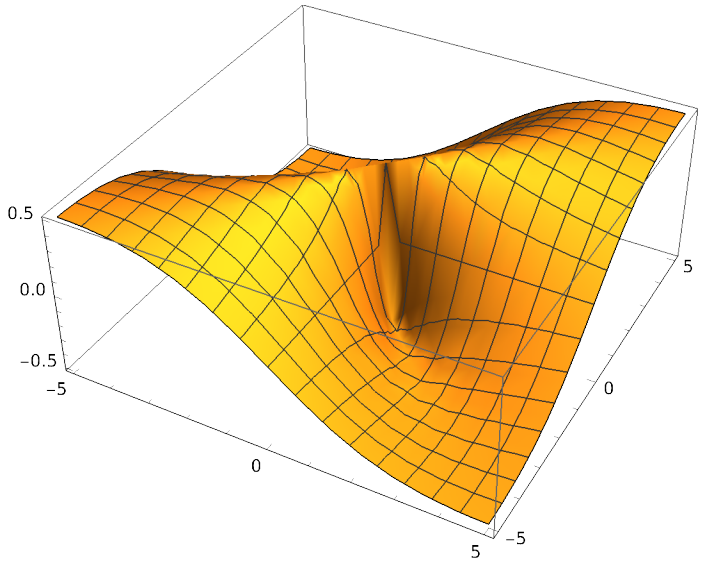
Out[34]=



In[35]:=
```
i[x_, y_] := x y / (x ^ 2 + y ^ 2)
```

In[36]:=
```
Plot3D[i[x, y], {x, -5, 5}, {y, -5, 5}]
```

Out[36]=



Q 2. Let f (x) = x / 1 + x ^ 2

In[37]:=
```
f[x] := x / (1 + x ^ 2)
```

a. Find f ' (x) and f " (x).

In[38]:=
```
D[f[x], x]
```

Out[38]=

$$-\frac{2\,x^2}{\left(1+x^2\right)^2}+\frac{1}{1+x^2}$$

In[39]:=
```
D[%, x]
```

Out[39]=
$$\frac{8\,x^3}{\left(1+x^2\right)^3}-\frac{6\,x}{\left(1+x^2\right)^2}$$

b. Find f ' (−1) and f ' (0).

In[40]:=
```
D[f[x], x] /. x → (-1)
```

Out[40]= 0

In[41]:=
```
D[f[x], x] /. x → 0
```

Out[41]= 1

c. Find f " (0) and f " (1).

In[42]:=
```
D[%, x] /. x → 0
```

Out[42]= 0

In[43]:=
```
D[%, x] /. x → 1
```

Out[43]= 0

In[44]:=
```
ClearAll[f]
```

Q 3. Find the prime factorization of each integer .

a. 3, 527, 218, 133, 309, 946, 276, 293.

In[45]:=
```
FactorInteger [3]
```

Out[45]= {{3, 1}}

In[46]:=
```
FactorInteger [527]
```

Out[46]= {{17, 1}, {31, 1}}

In[47]:=
```
FactorInteger [218]
```

Out[47]= {{2, 1}, {109, 1}}

In[48]:=
```
FactorInteger [133]
```

Out[48]= {{7, 1}, {19, 1}}

In[49]:=
```
FactorInteger [309]
```

Out[49]= {{3, 1}, {103, 1}}

In[50]:=
```
FactorInteger [946]
```

Out[50]= {{2, 1}, {11, 1}, {43, 1}}

In[51]:=
```
FactorInteger [276]
```

Out[51]= {{2, 2}, {3, 1}, {23, 1}}

In[52]:= **FactorInteger[293]**

Out[52]= {{293, 1}}

b. 471, 945, 325, 930, 166, 269

In[53]:= **FactorInteger[471]**

Out[53]= {{3, 1}, {157, 1}}

In[54]:= **FactorInteger[945]**

Out[54]= {{3, 3}, {5, 1}, {7, 1}}

In[55]:= **FactorInteger[325]**

Out[55]= {{5, 2}, {13, 1}}

In[56]:= **FactorInteger[930]**

Out[56]= {{2, 1}, {3, 1}, {5, 1}, {31, 1}}

In[57]:= **FactorInteger[166]**

Out[57]= {{2, 1}, {83, 1}}

In[58]:= **FactorInteger[269]**

Out[58]= {{269, 1}}

c. 471, 945, 325, 930, 166, 281

In[59]:= **FactorInteger[471]**

Out[59]= {{3, 1}, {157, 1}}

In[60]:= **FactorInteger[945]**

Out[60]= {{3, 3}, {5, 1}, {7, 1}}

In[61]:= **FactorInteger[325]**

Out[61]= {{5, 2}, {13, 1}}

In[62]:= **FactorInteger[930]**

Out[62]= {{2, 1}, {3, 1}, {5, 1}, {31, 1}}

In[63]:= **FactorInteger[166]**

Out[63]= {{2, 1}, {83, 1}}

In[64]:= **FactorInteger[281]**

Out[64]= {{281, 1}}

Q 4. Compute each expression .

a. 3^6 mod 7

In[65]:=
```
Mod[3^6, 7]
```
Out[65]= 1

b. 6^10 mod 11

In[66]:=
```
Mod[6^10, 11]
```
Out[66]= 1

c. 7^20 mod 21

In[67]:=
```
Mod[7^20, 21]
```
Out[67]= 7

d. 7^22 mod 23

In[68]:=
```
Mod[7^22, 23]
```
Out[68]= 1

Q 8. Let M = [{1, 1}, {1, 0}]

In[69]:=
```
M = {{1, 1}, {1, 0}}
```
Out[69]= {{1, 1}, {1, 0}}

a. Find M^2, M^3, ..., M^10

In[70]:=
```
MatrixPower[M, 2]
```
Out[70]= {{2, 1}, {1, 1}}

In[71]:=
```
MatrixPower[M, 3]
```
Out[71]= {{3, 2}, {2, 1}}

In[72]:=
```
MatrixPower[M, 4]
```
Out[72]= {{5, 3}, {3, 2}}

In[73]:=
```
MatrixPower[M, 5]
```
Out[73]= {{8, 5}, {5, 3}}

In[74]:=
```
MatrixPower[M, 6]
```
Out[74]= {{13, 8}, {8, 5}}

In[75]:=
```
MatrixPower[M, 7]
```
Out[75]= {{21, 13}, {13, 8}}

In[76]:=
```
MatrixPower[M, 8]
```
Out[76]= {{34, 21}, {21, 13}}

In[77]:=
```
MatrixPower[M, 9]
```
Out[77]= {{55, 34}, {34, 21}}

In[78]:=
```
MatrixPower[M, 10]
```
Out[78]= {{89, 55}, {55, 34}}

b. Find the 100 th Fibonnaci number.

In[79]:=
```
f[0] = 1;
```

In[80]:=
```
f[1] = 1;
```

In[81]:=
```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```

In[82]:=
```
f[100]
```
Out[82]= 573 147 844 013 817 084 101

Q9. Find solutions of following equations .

a. Find x, if x^2 + x = 1.

In[83]:=
```
Solve[x^2 + x == 1, x]
```
Out[83]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

b. Find x, x^2 + x = -1.

In[84]:=
```
Solve[x^2 + x == -1, x]
```
Out[84]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

c. Find x and y .

$$4x - 3y = 5$$
$$6x + 2y = 14$$

In[85]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```
Out[85]= {{x → 2, y → 1}}

d. Find x, y, z and t

$$-2x - 2y + 3z + t = 8$$
$$-3x + 0y - 6z + t = -19$$
$$6x - 8y + 6z + 5t = 47$$
$$x + 3y - 3z - t = -9$$

In[86]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```
Out[86]= {{x → 2, y → 1, z → 3, t → 5}}

Q10.

In[19]:=
```
FindRoot[250 * e^r + 300 * e^0.75 r + 350 * e^0.5 r + 400 * e^0.25 r == 1365, {r, 0}]
```

FindRoot : The function value $\{-1365 + 250\, e^{1.49012\times10^{-8}} + 5.96046\times10^{-6}\, e^{0.25} + 5.21541\times10^{-6}\, e^{0.5} + 4.47035\times10^{-6}\, e^{0.75}\}$ is not a list of numbers with dimensions $\{1\}$ at $\{r\} = \{1.49012\times10^{-8}\}$.

Out[19]=
```
{r → 0.}
```

Q 11.

In[20]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = g + (n / 2) / (2); i = i + 1];
  g]
```

In[21]:=
```
N[mysqrt[2], 6]
```

Out[21]=
```
11.0000
```

In[22]:=
```
N[sqrt[2], 6]
```

Out[22]=
```
sqrt[2.00000]
```

In[23]:=
```
N[mysqrt[3]]
```

Out[23]=
```
16.
```

Q12

In[24]:=
```
Clear[collatz];
```

In[6]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
  collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]]
```

In[7]:=
```
collatz[27]
```

Out[7]=
```
111
```