ASSIGNMENT-CHAPTER 12

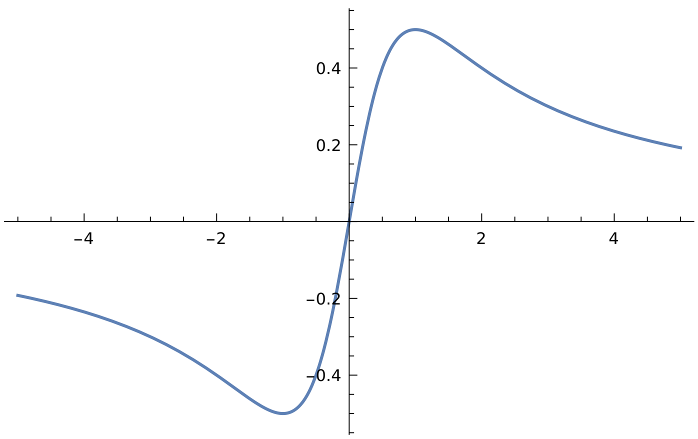
EXERCISES

1. Graph each of the functions. Experiment with different domains or viewpoints to display the best image.

(a) f(x) = x/1+x^2

In[1]:=
```
Plot[x / (1 + x ^ 2), {x, -5, 5}]
```

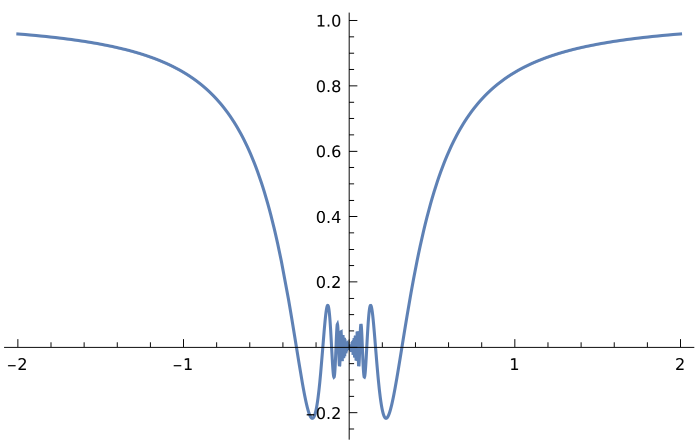
Out[1]=



(b) y = x sin(1/x)

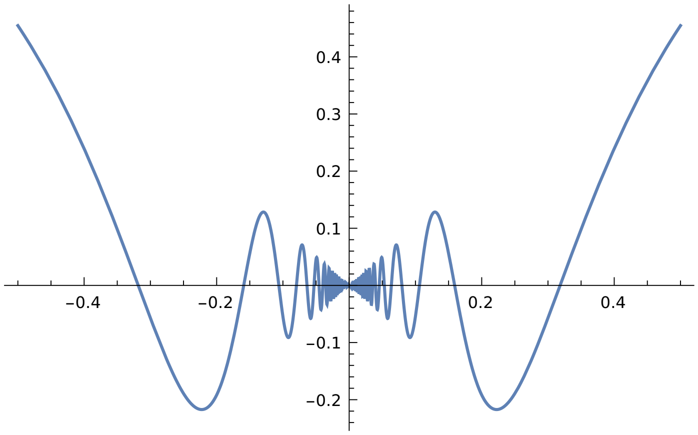
In[2]:=
```
Plot[x Sin[1 / x], {x, -2, 2}]
```

Out[2]=



In[3]:= `Plot[x Sin[1 / x], {x, -0.5, 0.5}]`

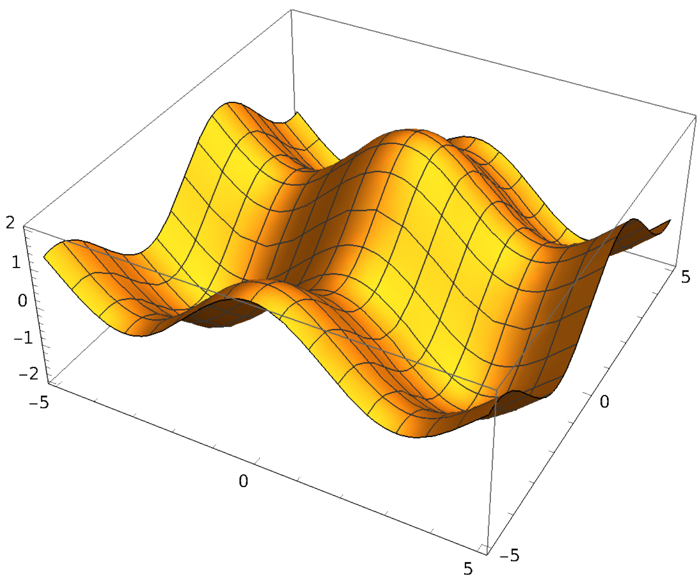
Out[3]=



(c) g(x,y) = cos(x)+sin(y)

In[4]:= `Plot3D[Cos[x] + Sin[y], {x, -5, 5}, {y, -5, 5}]`

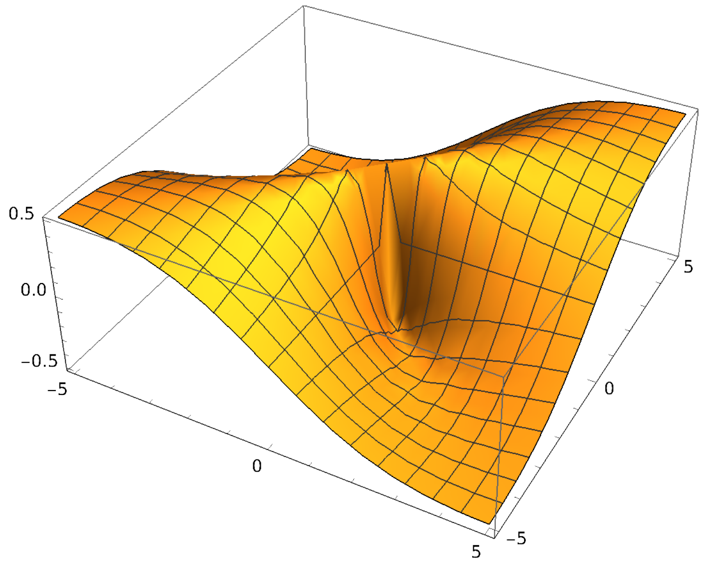
Out[4]=



(d) z = x y/x^2+y^2

In[5]:= `Plot3D[x y / (x ^ 2 + y ^ 2), {x, -5, 5}, {y, -5, 5}]`

Out[5]=



2. Let f(x) = x/(1+x^2) .

(a) Find f '(x) and f " (x).

In[6]:= `f[x_] := x / (1 + x ^ 2)`

In[7]:= `f '[x]`

Out[7]= $-\frac{2 x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[8]:= `f ''[x]`

Out[8]= $\frac{8 x^3}{(1+x^2)^3}-\frac{6 x}{(1+x^2)^2}$

(b) Find f '(-1) and f ' (0)

In[9]:= `f '[-1]`

Out[9]= 0

In[10]:= `f '[0]`

Out[10]= 1

(c) Find f''(0) and f''(1)

In[11]:= `f ''[0]`

Out[11]= 0

In[12]:=
```
f''[1]
```
Out[12]= $-\frac{1}{2}$

3. Find the prime factorisation of each integer.

(a) 3,527,218,133,309,949,276,293

In[13]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```
Out[13]= {{15 013 , 2}, {25 013 , 3}}

(b) 471,945,325,930,166,269

In[14]:=
```
FactorInteger [ 471 945 325 930 166 269 ]
```
Out[14]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

(c) 471,945,325,930,166,281

In[15]:=
```
FactorInteger [ 471 945 325 930 166 281 ]
```
Out[15]= {{471 945 325 930 166 281 , 1}}

4. Compute each expression .

(a) 3^6 mod 7

In[16]:=
```
Mod[3 ^ 6, 7]
```
Out[16]= 1

(b) 6^10 mod 11

In[17]:=
```
Mod[6 ^ 10, 11]
```
Out[17]= 1

(c) 7^20 mod 21

In[18]:=
```
Mod[7 ^ 20, 21]
```
Out[18]= 7

(d) 7^22 mod 23

In[19]:=
```
Mod[7 ^ 22, 23]
```
Out[19]= 1

8. Let M = $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) Find M^2, M^3, ... ,M^10.

In[20]:=
```
M = {{1, 1}, {1, 0}};
M // MatrixForm
```
Out[21]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

In[22]:=
```
M.M;
M.M // MatrixForm
```
Out[23]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[24]:=
```
M3 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
M3 // MatrixForm
```
Out[25]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[26]:=
```
M4 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
M4 // MatrixForm
```
Out[27]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[28]:=
```
M10 = MatrixPower [{{1, 1}, {1, 0}}, 10];
M10 // MatrixForm
```
Out[29]//MatrixForm=

$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

(b) Do your answers suggest a way to compute Fibonacci numbers? Find the 100th Fibonacci number.

In[30]:=
```
M100 = MatrixPower [{{1, 1}, {1, 0}}, 100];
M100 // MatrixForm
```
Out[31]//MatrixForm=

$$\begin{pmatrix} 573\,147\,844\,013\,817\,084\,101 & 354\,224\,848\,179\,261\,915\,075 \\ 354\,224\,848\,179\,261\,915\,075 & 218\,922\,995\,834\,555\,169\,026 \end{pmatrix}$$

Yes, by finding matrix power of M, we get more efficient way to compute fibonacci numers .Thus the first elememt of M100 matrix is 100th Fibonacci number, that is 573147844013817084101 .

9. Find solutions to the following equations or system of equations .

(a) Find x, if x^2+x=1.

In[32]:=
```
Solve[x ^ 2 + x == 1, x]
```
Out[32]=

$$\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$$

(b) Find x, if x^2+x=-1.

In[33]:=
```
Solve[x ^ 2 + x == -1, x]
```

Out[33]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) Find x and y.

4 x-3 y=5

6 x+2 y=14

In[34]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```

Out[34]= {{x → 2, y → 1}}

(d) Find x ,y ,z and t.

-2 x-2 y+3 z+t=8

-3 x+0 y-6 z+t=-19

6 x-8 y+6 z+5 t=47

x+3 y-3 z-t=-9

In[35]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8 , -3 x + 0 y - 6 z + t == -19,
   6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[35]= {{x → 2, y → 1, z → 3, t → 5}}

10. Solve this equation for r: 250 ⅇ^(1.0 r)+300ⅇ^(0.75 r)+350ⅇ^(0.5 r)+400 ⅇ^(0.25 r)==1365.

In[36]:=
```
FindRoot[{250 ⅇ ^ (1.0 r) + 300 ⅇ ^ (0.75 r) + 350 ⅇ ^ (0.5 r) + 400 ⅇ ^ (0.25 r) == 1365}, {r, 0}]
```

Out[36]= {r → 0.084104}

11. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1 ,finds 20 new guesses.

In[37]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + (n / g)) / 2; i = i + 1]; g]
```

In[38]:=
```
N[mysqrt[2], 6]
```

Out[38]= 1.41421

In[39]:=
```
N[mysqrt[2], 6]
```

Out[39]= 1.41421

In[40]:=
```
N[mysqrt[3]]
```

Out[40]= 1.73205

12. (a) Write a (recursive) function called collatz that accepts a single argument, n and returns:

- 0 if n is equal to 1
- 1+collatz(n/2) if n is even
- 1+collatz(3*n+1) if n is odd

In[41]:=
```
Clear[collatz]
```

In[42]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
  collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]]
```

(b) Find the values for n= 1, 2

In[43]:=
```
collatz[1]
```

Out[43]= 0

In[44]:=
```
collatz[2]
```

Out[44]= 1