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WEYL-TYPE THEOREMS FOR ADJOINTS OF UNBOUNDED OPERATORS WITH ASCENT 0 OR 1

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ABSTRACT. If T is a densely defined closed linear operator with ascent $p(T-\lambda I) \leq 1$, for all complex numbers λ, then several Weyl-type theorems and their variants are equivalent for T^* and T^* satisfies each of these variants.

1. INTRODUCTION

In [12], Weyl asserts that if T is a bounded hermitian operator, then $\sigma_w(T)$ consists precisely of all points in $\sigma(T)$ except the isolated eigenvalues of finite multiplicity. Since then Weyl's Theorem has been extended to the class of bounded normal, hyponormal and Toeplitz operators [6], and to several other non-normal classes of bounded operators. Further in [3], Berkani proved that if T is a bounded normal operator acting on a Hilbert space H, then $\sigma_{BW}(T) = \sigma(T) \setminus E(T)$, where $E(T)$ is the set of all isolated eigenvalues of T. This gives the generalization of the classical Weyl's theorem. Also, in [4] he proved this generalized version of classical Weyl's theorem for bounded hyponormal operators. However, the study so far has been limited to the classes of bounded operators. Recently, we have extended this study to the class of *unbounded* Normal operator (communicated), class of *unbounded* Hyponormal operators [9] and other non-normal classes of operators possessing the common property that the ascent $p(T-\lambda I) \leqslant 1$, for every $\lambda \in \mathbb{C}$. Further in 1969, Berberian [2] proved the Weyl's theorem for T^* when T is a bounded hyponormal operator and Duggal and Kubrusly [7] proved that T^* satisfies a-Weyl's theorem when T is a bounded totally posinormal operator with the additional condition that $\operatorname{iso}\sigma(T) \cap \sigma_{kt}(T) = \emptyset$, where $\operatorname{iso}\sigma(T)$ is the set of isolated points in $\sigma(T)$ and $\sigma_{kt}(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is not Kato-type}\}$. Note that totally posinormal operators were called "conditionally totally posinormal" in [7].

Let H be an infinite dimensional complex Hilbert space and $B(H)$ and $C(H)$ denote the set of all bounded linear operators and closed linear operators from H to H, respectively. We define a class $\mathcal{N}$ of operators as $\mathcal{N} = \{T \in C(H) : \sigma(T|_M - \lambda I) = \{0\} \Longrightarrow (T|_M - \lambda I) = 0$ for every invariant subspace M of $T\}$.

In this paper, we denote by $\Theta(H) = \{T \in \mathcal{N} :$ domain $\mathfrak{D}(T)$ is dense in H and ascent $p(T-\lambda I) \leqslant 1,\ \forall \lambda \in \mathbb{C}$, with $\rho(T) \neq \emptyset\}$ and we study several Weyl-type theorems and properties for the adjoint T^*, whenever $T \in \Theta(H)$. The second section deals with the spectral theory for T^*, when $T \in \Theta(H)$. It is proved that an operator $T \in \Theta(H)$ is a polaroid

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