EMBEDDINGS AND C^*-ENVELOPES OF EXACT OPERATOR SYSTEMS

PREETI LUTHRA and AJAY KUMAR[⊠]

Abstract

We prove a necessary and sufficient condition for embeddability of an operator system into O_2. Using Kirchberg's theorems on a tensor product of O_2 and O_∞, we establish results on their operator system counterparts S_2 and S_∞. Applications of the results, including some examples describing C^*-envelopes of operator systems, are also discussed.

1. Introduction

Operator systems with universal generators for some well-studied C^*-envelopes have attracted considerable interest in recent years. Zheng [21] introduced the operator system S_n generated by Cuntz isometries and, later, in [17], Paulsen and Zheng explored tensor products and nuclearity for this operator system.

In 1977, Cuntz [5] introduced the C^*-algebras O_n ($1 \le n \le \infty$). These were the first explicit examples of simple infinite separable C^*-algebras. Cuntz proved that his algebras are simple and purely infinite and independent of the choice of generators.

These algebras played an important role in the classification theory of purely infinite, simple, separable and nuclear C^*-algebras, by Kirchberg and Philips. The classification theory for separable C^*-algebras with certain properties in terms of the Cuntz algebras O_2 and O_∞ was given by Kirchberg and Rørdam (see [19]).

Kirchberg established three fundamental theorems: the embedding of separable exact C^*-algebras into the Cuntz algebra O_2 and the tensor product theorems for O_2 and O_∞. Many generalisations were later proved by Kirchberg and Rørdam. Recently, Lupini [15] established an operator system analogue of Kirchberg's nuclear embedding theorem involving the Gurarij operator system $\mathbb{GS}$.

For $1 \le n \le \infty$, O_n is a simple C^*-algebra, so O_n is the C^*-envelope of S_n (see [21]). This motivates our study of Kirchberg's theorems on O_n ($2 \le n \le \infty$) in terms of the C^*-envelopes of operator systems.

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