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## EMBEDDINGS AND C\*-ENVELOPES OF EXACT OPERATOR SYSTEMS

## PREETI LUTHRA and AJAY KUMAR™

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## Abstract

We prove a necessary and sufficient condition for embeddability of an operator system into  $O_2$ . Using Kirchberg's theorems on a tensor product of  $O_2$  and  $O_\infty$ , we establish results on their operator system counterparts  $S_2$  and  $S_\infty$ . Applications of the results, including some examples describing  $C^*$ -envelopes of operator systems, are also discussed.

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## 1. Introduction

Operator systems with universal generators for some well-studied  $C^*$ -envelopes have attracted considerable interest in recent years. Zheng [21] introduced the operator system  $S_n$  generated by Cuntz isometries and, later, in [17], Paulsen and Zheng explored tensor products and nuclearity for this operator system.

In 1977, Cuntz [5] introduced the  $C^*$ -algebras  $O_n$  ( $1 \le n \le \infty$ ). These were the first explicit examples of simple infinite separable  $C^*$ -algebras. Cuntz proved that his algebras are simple and purely infinite and independent of the choice of generators.

These algebras played an important role in the classification theory of purely infinite, simple, separable and nuclear  $C^*$ -algebras, by Kirchberg and Philips. The classification theory for separable  $C^*$ -algebras with certain properties in terms of the Cuntz algebras  $O_2$  and  $O_\infty$  was given by Kirchberg and Rørdam (see [19]).

Kirchberg established three fundamental theorems: the embedding of separable exact  $C^*$ -algebras into the Cuntz algebra  $O_2$  and the tensor product theorems for  $O_2$  and  $O_{\infty}$ . Many generalisations were later proved by Kirchberg and Rørdam. Recently, Lupini [15] established an operator system analogue of Kirchberg's nuclear embedding theorem involving the Gurarij operator system  $\mathbb{GS}$ .

For  $1 \le n \le \infty$ ,  $O_n$  is a simple  $C^*$ -algebra, so  $O_n$  is the  $C^*$ -envelope of  $S_n$  (see [21]). This motivates our study of Kirchberg's theorems on  $O_n$  ( $2 \le n \le \infty$ ) in terms of the  $C^*$ -envelopes of operator systems.

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