Operators and Matrices
Volume 11, Number 2 (2017), 587–592 doi:10.7153/oam-11-38

ISOMETRIC COMPOSITION OPERATORS ON THE FOCK–SPACES

ANURADHA GUPTA AND POOJA SHARMA

(*Communicated by R. Curto*)

Abstract. In this paper a necessary and sufficient condition for a holomorphic self map ϕ on $\mathbb{C}^N$ to induce an isometric composition operator on the Fock space has been obtained. Some necessary and sufficient conditions for a composition operator C_ϕ to be a quasi-isometric and m-isometric have also been explored.

1. Introduction

Let $z=(z_1,z_2,\dots,z_N)$ and $w=(w_1,w_2,\dots,w_N)$ be points in $\mathbb{C}^N$, $\langle z,w\rangle = \sum_{k=1}^{N} z_k\overline{w_k}$ and $|z| = \sqrt{\langle z,z\rangle}$. Let $\mathbb{B}$ denote the open unit ball $\{z : |z|<1\}$, $S=\partial\mathbb{B}$ the boundary of the unit ball $\mathbb{B}$, $dm(z)=rdrd\theta$ the Lebesgue area measure on $\mathbb{C}$, $dV(z)$ the Lebesgue volume measure on $\mathbb{C}^N$, $V_N=V(\mathbb{B})$, $d\sigma(z)$ the normalized surface measure on S and $H(\mathbb{C}^N)$ the space of all holomorphic functions on $\mathbb{C}^N$ (entire functions). For $p,\alpha\in(0,\infty)$, the Bergman-Fock space[22] $\mathscr{F}^p_\alpha=\mathscr{F}^p_\alpha(\mathbb{C}^N)$ is the space of all entire functions f for which

Self-attested
Pooja

$$\|f\|^p_{\mathscr{F}^p_\alpha} = \left(\frac{p\alpha}{2\pi}\right)^N \int_{\mathbb{C}^N} |f(z)|^p e^{-\frac{p\alpha}{2}|z|^2}dV(z) < \infty$$

Note that, by using polar coordinates

$$\begin{aligned}\|1\|^p_{\mathscr{F}^p_\alpha} &= \left(\frac{p\alpha}{2\pi}\right)^N V_N \int_0^\infty \int_S \rho^{2N-1}e^{-\frac{\alpha p}{2}\rho^2}d\sigma(\xi)d\rho \\ &= \frac{(p\alpha)^N\int_0^\infty t^{N-1}e^{-\frac{\alpha p}{2}t}dt}{2^N(N-1)!} = 1.\end{aligned}$$

When $1\leqslant p<\infty$, the space $\mathscr{F}^p_\alpha(\mathbb{C}^N)$ is a Banach space with the norm $\|f\|_{\mathscr{F}^p_\alpha}$, while for $p\in(0,1)$, it is an F-space with the translation-invariant metric $d_{\mathscr{F}^p_\alpha}(f,g) = \|f-g\|^p_{\mathscr{F}^p_\alpha}$.

For $p=2$ the space is reduced to the Fock space, which is a functional Hilbert space with the inner product

$$\langle f,g\rangle = \left(\frac{\alpha}{\pi}\right)^N \int_{\mathbb{C}^N} f(z)\overline{g(z)}e^{-\alpha|z|^2}dV(z).$$

Mathematics subject classification (2010): 47B33, 46B04, 30H20.
Keywords and phrases: Composition operators, isometry, Fock spaces.