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## NUCLEARITY PROPERTIES AND $C^*$ -ENVELOPES OF OPERATOR SYSTEM INDUCTIVE LIMITS

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**ABSTRACT.** We investigate the relationship between  $C^*$ -envelopes and inductive limit of operator systems. Various operator system nuclearity properties of inductive limit for a sequence of operator systems are also discussed.

### 1. Introduction

In last few years, the development of the theory of operator systems has seen a fair amount of attention. All the important notions from the theory of  $C^*$ -algebras including exactness, nuclearity, weak expectation property and lifting properties have been explicitly defined in the category of operator systems. Associated to every representation  $\phi$  of an operator system  $\mathcal{S}$  into  $C^*$ -algebra of bounded operator  $B(H)$ , for some Hilbert Space  $H$ , one can always consider a  $C^*$ -cover generated by  $\phi(\mathcal{S})$  that is, the  $C^*$ -algebra  $C^*(\phi(\mathcal{S})) \subset B(H)$ . The minimal  $C^*$ -cover among all such representations is known as the  $C^*$ -envelope of  $\mathcal{S}$ . It is thus quite natural to ask which  $C^*$ -algebraic properties of the  $C^*$ -envelopes are carried over to the generating operator systems in terms of their definitions in the operator system category, and to what extent. Some attempts done in this direction can be found in [5, 12].

It is well known (see [2]) that for the category of  $C^*$ -algebras, inductive limit preserves many intrinsic properties, viz., exactness, nuclearity, simplicity etc. The analysis of inductive limit of ascending sequences of finite dimensional  $C^*$ -algebras, known as approximately finite dimensional (AF)  $C^*$ -algebras, has played an important role in theory of operator algebras. Existence of inductive limits in the category of operator systems has been shown in [10]. But unlike in the category of  $C^*$ -algebras, there are several notion of nuclearity in the operator system category (see [7–9]). It is thus natural to check if these nuclearity properties are preserved under operator system inductive limit. This

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