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CHARACTERIZATION OF EXTENDED HAMMING AND GOLAY CODES AS PERFECT CODES IN POSET BLOCK SPACES

B. K. Dass and Namita Sharma*

Department of Mathematics, University of Delhi
Delhi-110 007, India

Rashmi Verma

Mata Sundri College for Women (University of Delhi)
Mata Sundri Lane, Delhi-110 002, India

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Abstract. Alves, Panek and Firer (Error-block codes and poset metrics, *Adv. Math. Commun.*, 2 (2008), 95–111) classified all poset block structures which turn the [8, 4, 4] extended binary Hamming code into a 1-perfect poset block code. However, the proof needs corrections that are supplied in this paper. We provide a counterexample to show that the extended binary Golay code is not 1-perfect for the proposed poset block structures. All poset block structures turning the extended binary and ternary Golay codes into 1-perfect codes are classified.

1. Introduction

Most of the studies in coding theory have been made in finite-dimensional vector spaces $\mathbb{F}_q^n$ over a finite field $\mathbb{F}_q$ and equipped with a metric, the most common one being the Hamming metric. The error-correction capability of a code is largely determined by its minimum distance. Finding, for integers $n \geq k \geq 1$, a k-dimensional subspace of $\mathbb{F}_q^n$ (called a "linear code of length n over F_q") with the largest possible minimum distance is one of the main problems of coding theory. For linear codes in the Hamming space, this problem has an equivalent formulation in terms of the parity-check matrix of the code, as is well known. The resulting problem for matrices was generalized by Niederreiter [20, 21]. With this generalization as basis and using the concept of partially ordered sets, Brualdi, Graves and Lawrence [4] in 1995 introduced the concept of codes with a poset metric. Poset metrics gave rise to many new perfect codes [1, 6, 15, 17], a fact attributed to the increased packing radius with respect to a poset metric. The [8, 4, 4] extended binary Hamming code, the [24, 12, 8] extended binary Golay code and the [12, 6, 6] extended ternary Golay code, though not perfect with respect to the Hamming metric, turn out to be perfect with respect to certain poset metrics. For the construction and uniqueness of the codes mentioned above, see [14].

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* Corresponding author: Namita Sharma.

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