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REPEATED BURST ERROR CORRECTING LINEAR CODES

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Many kinds of errors in coding theory have been dealt with for which codes have been constructed to combat such errors. Though there is a long history concerning the growth of the subject and many of the codes developed have found applications in numerous areas of practical interest, one of the areas of practical importance in which a parallel growth of the subject took place is that of burst error detecting and correcting codes. The nature of burst errors differ from channel to channel depending upon the behaviour of channels or the kind of errors which occur during the process of data transmission. In very busy communication channels, errors repeat themselves more frequently. In view of this, it is desirable to consider repeated burst errors. The paper presents lower and upper bounds on the number of parity-check digits required for a linear code correcting errors in the form of repeated bursts. An upper bound for a code that detects m-repeated bursts has also been derived. Illustrations of several codes that correct 2-repeated bursts of different lengths have also been given.

Keywords: Linear codes; burst errors; repeated bursts.

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1. Introduction

In the theory of error correcting codes, there are many kinds of errors for which codes have been developed to detect and correct such errors. Codes developed at the early stages were meant mainly to detect and correct random errors. However, it was noticed later that in many channels, the likelihood of the occurrence of errors was more in adjacent positions rather than their occurrence in a random manner. It was in this spirit that codes correcting single errors and double adjacent errors were developed by Abramson [1]. Upon generalization of this idea, such errors were put in the category of errors called burst errors. It has been noticed that in many channels, errors occur predominantly in the form of bursts. Since the development of various burst error correcting and detecting codes, several variants and modifications of the definition of a burst error came up depending upon the various kinds of channels which were in use. A burst of length b may be defined as follows:

Definition 1.1. A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last component of which is non-zero.

This definition was given by P. Fire [6] in a research report wherein he called such errors as 'open-loop burst errors' and in the same report he considered another kind of burst errors as well which were called 'closed-loop burst errors'. A closed-loop burst of length b is defined as follows:

Definition 1.2. Let b be an integer and $x = (\xi_1, \xi_2, \ldots, \xi_n)$ be a vector in $V^n(q)$, a vector space of n-tuples over GF(q). If $2 \leq b \leq \frac{(n+1)}{2}$, then x is called a 'closed-loop burst vector of length b' whenever there is an i such that $1 \leq i \leq b-1, \xi_i \cdot \xi_{n-b+i+1} \neq 0, \xi_{i+1} = \xi_{i+2} = \ldots = \xi_{n-b+i} = 0$.

There is yet another kind of burst error due to Chien and Tang [4] which has drawn attention of many researchers as follows:

Definition 1.3. A burst of length b is a vector whose only non-zero components are confined to some b consecutive components, the first of which is non-zero.

Still amongst the various generalizations of bursts, there are codes which have been developed for the correction of multiple bursts, bursts of bursts, random multiple bursts, low-density bursts etc. A good deal of research work has been devoted for the development of burst/multiple burst correcting codes (see [15], [14], [8], [12], [3], [16] and others). Amongst the category of multiple bursts, there is no uniform terminology. Looking at the fact that there are many channels which require development of codes dealing with multiple bursts (the channels studied in [10] fall in this category), we in this paper propose a new kind of burst error which will be called repeated burst error. Such an error may be defined as follows:

Definition 1.4. An m-repeated burst of length b is a vector of length n whose only non-zero components are confined to m distinct sets of b consecutive components, the first and the last component of each set being non-zero.

The development of such codes detecting and correcting such errors may prove to be useful for channels already dealing with multiple burst errors thereby improving upon their efficiency. Such codes emerge as a natural generalization of single burst error correcting and detecting codes.

In particular, a 2-repeated burst may be defined as follows:

Definition 1.5. A 2-repeated burst of length b is a vector of length n whose only non-zero components are confined to two distinct sets of b consecutive components, the first and the last components of each set being non-zero.

As an illustration (0012040003012000) is a 2-repeated burst of length 4 over GF(5).

Consider a 2-repeated burst of length b. If the first non-zero component is the ith component, $i = 1, 2, \ldots, n-2b+1$, then the first burst may go upto $(i+b-1)$th components and the second burst is in the remaining last $(n-i-b+1)$ components. However, for $i = n-2b+2, \ldots, n$ (i.e. in the last $2b-1$ components) any number of non-zero components will be considered as a 2-repeated burst of length b or less.

The paper has been organised as follows. In Section 2, we derive a bound analogus to Fire's bound (see [6]; [11, Theorem 4.16]) for correction of 2-repeated burst of length b or less and then obtain in the following result another bound proving the existence of such a code. Section 3 describes bounds for m-repeated burst errors. Firstly, we obtain a necessary condition for the correction of m-repeated burst error correcting code followed by a sufficient condition for the existence of such a code. The last section, namely section 4 presents yet another bound for codes detecting m-repeated burst errors. The paper concludes with an illustration of a 2-repeated burst of length 3 or less correcting code and another illustration of a 2-repeated burst of length b or less correcting code over GF(2).

In what follows, an (n, k) linear code will be considered as a subspace of the space of all n-tuples over GF(q). The distance between two vectors shall be considered in the Hamming sense.

2. Correction of 2-Repeated Bursts

In this section, we consider linear codes capable of correcting 2-repeated bursts of length b or less. In the following, we obtain a necessary condition for the existence of such a code. The technique followed is that of the well-known Hamming's sphere-packing bound (see [7]; [11, Theorem 4.5]) of enumerating the total number of correctable error vectors and then comparing it with the total number of cosets.

Before we prove the main results, we give below an extension of problem 2.2(b) [9] a result which will be used in proving subsequent results of this paper. The problem stated in [9] is as follows:

Problem. Show that

$$\sum_{k=0}^{n-1} \binom{2+k}{2} = \binom{2+n}{3}$$

and thus

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n+1) = \frac{1}{3} n(n+1)(n+2).$$

This result can be extended as follows:

$$\sum_{i=r}^{n}\binom{i}{r} = \binom{r}{r} + \binom{r+1}{r} + \ldots + \binom{n}{r} = \binom{n+1}{r+1}. \tag{2.1}$$

Proof. We prove the result by induction on n.

For $n = 1, r$ can only be 1.

L.H.S. = R.H.S. = 1.

Therefore the result holds for $n = 1$.

Assume the result to be true for $n = p$.

For $n = p + 1$,

$$\begin{aligned}
&\binom{r}{r} + \binom{r+1}{r} + \ldots + \binom{p+1}{r} \\
&= \left(\binom{r}{r} + \binom{r+1}{r} + \ldots \binom{p}{r}\right) + \binom{p+1}{r} \\
&= \binom{p+1}{r+1} + \binom{p+1}{r} \text{ (by induction hypothesis)} \\
&= \binom{p+2}{r+1} \text{ (using the identity} \binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}).
\end{aligned}$$

Therefore the result holds for $n = p + 1$.

Hence, by the principle of mathematical induction, (2.1) is true for every positive integer n. □

Alternatively, the above result may also be derived as follows:

Alternate Proof. Consider the finite geometric series

$$(1 + x) + (1 + x)^2 + (1 + x)^3 + \cdots + (1 + x)^n . \tag{2.2}$$

The coefficient of x^r in (2.2) is

$$\binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \cdots + \binom{n}{r}. \tag{2.3}$$

Also the sum of the series (2.2) is

$$(1 + x)\left(\frac{(1 + x)^n - 1}{(1 + x) - 1}\right) = \frac{(1 + x)^{n+1} - (1 + x)}{x}$$

which gives the coefficient of x^r as

$$\binom{n+1}{r+1}. \tag{2.4}$$

From (2.3) and (2.4), we get

$$\sum_{i=r}^{n}\binom{i}{r} = \binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \cdots + \binom{n}{r} = \binom{n+1}{r+1}. \qquad \square$$

Theorem 2.1. *Any* (n,k) *linear code over* $\mathrm{GF}(q)$ *that corrects all* 2*-repeated bursts of length* b *or less must satisfy*

$$q^{n-k} \geqslant q^{2(b-1)} \left(\binom{n-2b+2}{2}(q-1)^2 + \binom{n-2b+1}{1}(q-1) + q \right). \tag{2.5}$$

Proof. We first enumerate the total number of correctable error vectors which are 2-repeated bursts of length b or less.

Consider a 2-repeated burst of length b or less. Let the first non-zero component of the vector be the ith component which may go upto the $(i+b-1)$th component, where $i = 1, 2, \ldots, n-2b+1$. The second vector of length b or less will then be in the last (remaining) $(n-b-i+1)$ components. From Fire's bound (see [6]; [11, Theorem 4.16]), the number of bursts of length b or less in a vector of length $(n-b-i+1)$ is

$$\begin{aligned} &q^{b-1} \left(\binom{n-b-i+1-b+1}{1}(q-1) + 1 \right) \\ &= q^{b-1} \left(\binom{n-2b-i+2}{1}(q-1) + 1 \right). \end{aligned}$$

Summing over i gives the number of 2-repeated burst vectors with the first non-zero component in the first $(n-2b+1)$ components as

$$\begin{aligned} &\sum_{i=1}^{n-2b+1} (q-1)q^{b-1} \left[q^{b-1} \left(\binom{n-2b-i+2}{1}(q-1) + 1 \right) \right] \\ &= q^{2(b-1)} \left(\sum_{i=1}^{n-2b+1} \binom{n-2b-i+2}{1}(q-1)^2 + \sum_{i=1}^{n-2b+1} (q-1) \right) \\ &= q^{2(b-1)} \left(\binom{n-2b+2}{2}(q-1)^2 + \binom{n-2b+1}{1}(q-1) \right) \qquad \text{(using (2.1))}. \end{aligned}$$

Also, the number of 2-repeated bursts of length b or less in the last $2b-1$ components is clearly q^{2b-1}. Thus, the total number of 2-repeated bursts of length b or less, including the vector of all zeros, is

$$\begin{aligned} &q^{2(b-1)} \left(\binom{n-2b+2}{2}(q-1)^2 + \binom{n-2b+1}{1}(q-1) \right) + q^{2b-1} \\ &= q^{2(b-1)} \left(\binom{n-2b+2}{2}(q-1)^2 + \binom{n-2b+1}{1}(q-1) + q \right). \end{aligned}$$

Since all such vectors being correctable error vectors must belong to distinct cosets of the standard array and the total number of cosets available equals q^{n-k}, we must have

$$q^{n-k} \geqslant q^{2(b-1)} \left(\binom{n-2b+2}{2}(q-1)^2 + \binom{n-2b+1}{1}(q-1) + q \right). \qquad \square$$

In the following result, we derive a sufficient condition on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varsharmov-Gilbert-Sacks bound by constructing a parity-check matrix for the desired code (see [11, Theorem 4.17]). This technique not only ensures the existence of such a code but also gives a method for constructing such codes.

Theorem 2.2. *There shall always exist an* (n, k) *linear code over* GF(q) *that corrects all* 2*-repeated bursts of length* b *or less* $(n > 4b)$ *provided that*

$$q^{n-k} > q^{4(b-1)}\left(\binom{n-4b+3}{3}(q-1)^3 + \binom{n-4b+2}{2}(q-1)^2 + \binom{n-4b+1}{1}(q-1)q + q^2\right). \tag{2.6}$$

Proof. In order to prove this result, we first prove the following lemma.

Lemma 2.1. *The number of* 3*-repeated bursts of length* b *or less in a vector of length* n *is*

$$q^{3(b-1)}\left(\binom{n-3b+3}{3}(q-1)^3 + \binom{n-3b+2}{2}(q-1)^2 + \binom{n-3b+1}{1}(q-1)q + q^2\right). \tag{2.7}$$

Proof. Consider a 3-repeated burst of length b or less. Let the first non-zero component of the vector be the ith component and such a burst may go upto the $(i+b-1)$th component, where $i = 1, 2, \ldots, n-3b+1$. The second and the third bursts of length b or less will then be in the last (remaining)$(n-b-i+1)$ components. To enumerate the number of second and third bursts of length b or less is equivalent to enumerating the number of 2-repeated bursts of length b or less in a vector of length $(n-b-i+1)$, which from Theorem 2.1 is

$$q^{2(b-1)}\left(\binom{n-b-i+1-2b+2}{2}(q-1)^2 + \binom{n-b-i+1-2b+1}{1}(q-1) + q\right)$$
$$= q^{2(b-1)}\left(\binom{n-3b-i+3}{2}(q-1)^2 + \binom{n-3b-i+2}{1}(q-1) + q\right).$$

Summing on i gives the number of 3-repeated bursts of length b or less with the first non-zero component in the first $n-3b+1$ components as

$$\sum_{i=1}^{n-3b+1}(q-1)q^{b-1}\left[q^{2(b-1)}\left(\binom{n-3b-i+3}{2}(q-1)^2 + \binom{n-3b-i+2}{1}(q-1)+q\right)\right]$$

$$= q^{3(b-1)}\left(\sum_{i=1}^{n-3b+1}\binom{n-3b+3-i}{2}(q-1)^3 + \sum_{i=1}^{n-3b+1}\binom{n-3b+2-i}{1}(q-1)^2 + \sum_{i=1}^{n-3b+1}(q-1)q\right)$$

$$= q^{3(b-1)}\left(\binom{n-3b+3}{3}(q-1)^3 + \binom{n-3b+2}{2}(q-1)^2 + \binom{n-3b+1}{1}(q-1)q\right) \qquad \text{(using (2.1))}.$$

Also, the number of 3-repeated bursts of length b or less in the last $3b-1$ components is clearly q^{3b-1}.

Thus the total number of 3-repeated bursts of length b or less in a vector of length n, including the vector of all zeros, is

$$q^{3(b-1)}\left(\binom{n-3b+3}{3}(q-1)^3 + \binom{n-3b+2}{2}(q-1)^2 + \binom{n-3b+1}{1}(q-1)q\right) + q^{3b-1}$$

$$= q^{3(b-1)}\left(\binom{n-3b+3}{3}(q-1)^3 + \binom{n-3b+2}{2}(q-1)^2 + \binom{n-3b+1}{1}(q-1)q + q^2\right)$$

which proves the lemma. □

Proof of the theorem. We shall prove the result by constructing an appropriate $(n-k)\times n$ parity-check matrix $H=[h_1,h_2,\dots,h_n]$ for the desired code. Choose any non-zero $(n-k)$-tuple as the first column h_1 of H. Suppose that we have selected the first $j-1$ columns $h_1,h_2,\dots,h_{j-1}$ of H suitably. We lay down the condition to add the jth column h_j as follows:

h_j should not be a linear combination of the immediately preceding $(b-1)$ or fewer columns $h_{j-b+1},\dots,h_{j-1}$ of H together with any three distinct sets of b or fewer consecutive columns each from amongst the first $(j-b)$ columns $h_1,h_2,\dots,h_{j-b}$.

In other words,

$$\begin{aligned} h_j \neq\ & (\alpha_1 h_{j-b+1} + \alpha_2 h_{j-b+2} + \ldots + \alpha_{b-1} h_{j-1}) \\ & + (\beta_1 h_{i_1} + \beta_2 h_{i_1+1} + \ldots + \beta_b h_{i_1+b-1}) \\ & + (\gamma_1 h_{i_2} + \gamma_2 h_{i_2+1} + \ldots + \gamma_b h_{i_2+b-1}) \\ & + (\delta_1 h_{i_3} + \delta_2 h_{i_3+1} + \ldots + \delta_b h_{i_3+b-1}) \end{aligned}$$

where

$$\alpha_i, \beta_i, \gamma_i, \delta_i \in \mathrm{GF}(q), \qquad i_1 + i_2 + i_3 + 3b - 3 \leq j - b\,. \tag{2.8}$$

This condition ensures that there would not be a code-word which is expressible as a sum or difference of two vectors each of which is a 2-repeated burst of length b or less.

We now enumerate all possible linear combinations on the R.H.S. of (2.8).

Clearly, the coefficients α_i's which are $(b-1)$ in number can be chosen in $q^{(b-1)}$ ways.

To compute the coefficients β_i's, γ_i's and δ_i's is equivalent to find the number of 3-repeated bursts of length b or less in a vector of length $j-b$, which by Lemma 2.1 is

$$\begin{aligned} q^{3(b-1)} \Bigg(& \binom{j-4b+3}{3}(q-1)^3 + \binom{j-4b+2}{2}(q-1)^2 \\ & + \binom{j-4b+1}{1}(q-1)q + q^2 \Bigg)\,. \end{aligned}$$

Thus, the total number of linear combinations on the R.H.S. of (2.8) equals

$$\begin{aligned} & q^{b-1}\Bigg[q^{3(b-1)} \Bigg(\binom{j-4b+3}{3}(q-1)^3 + \binom{j-4b+2}{2}(q-1)^2 \\ & + \binom{j-4b+1}{1}(q-1)q + q^2 \Bigg)\Bigg] \\ & = q^{4(b-1)} \Bigg(\binom{j-4b+3}{3}(q-1)^3 + \binom{j-4b+2}{2}(q-1)^2 \\ & \quad + \binom{j-4b+1}{1}(q-1)q + q^2 \Bigg)\,. \end{aligned} \tag{2.9}$$

Since these many linear combinations cannot be put to be equal to h_j and at worst all the linear combinations computed in (2.9) might yield a distinct sum, therefore in view of the fact that the total number of $(n-k)$-tuples is q^{n-k}, the jth column h_j can be added to H provided that $q^{n-k} > (2.9)$.

To obtain a code of length n, replace j by n in the above inequality and we get

$$\begin{aligned} q^{n-k} > q^{4(b-1)} \Bigg(& \binom{n-4b+3}{3}(q-1)^3 + \binom{n-4b+2}{2}(q-1)^2 \\ & + \binom{n-4b+1}{1}(q-1)q + q^2 \Bigg)\,. \end{aligned}$$

□

3. Correction of m-Repeated Burst Errors

In this section, we first consider linear codes capable of correcting m-repeated bursts of length b or less. In the following, we obtain a necessary condition for the existence of such a code. The technique followed is that of the well-known Hamming's sphere-packing bound (see [7]; [11, Theorem 4.5]) as used in Theorem 2.1.

Theorem 3.1. *Any* (n, k) *linear code over* $\mathrm{GF}(q)$ *that corrects all* m*-repeated bursts of length* b *or less must satisfy*

$$q^{n-k} \geqslant q^{m(b-1)}\left(\binom{n-mb+m}{m}(q-1)^m + \sum_{l=0}^{m-1}\binom{n-mb+l}{l}(q-1)^l q^{m-1-l}\right). \quad (3.1)$$

Proof. We first enumerate the total number of correctable error vectors which are m-repeated bursts of length b or less.

We prove the result by induction on m applied to the R.H.S. of inequality (3.1).

For $m = 1$, expression on the R.H.S. of (3.1) becomes

$$q^{(b-1)}\left(\binom{n-b+1}{1}(q-1)+1\right)$$

which is the number of bursts of length b or less in a vector of length n as given in Fire's bound (see [6]; [11, Theorem 4.16]). So the result holds for $m = 1$.

Assume it to be true for $m = p$. Now we prove the result for $m = p+1$. Consider a $(p+1)$-repeated burst of length b or less. Let the first non-zero component of the vector be the ith component which may go upto the $(i+b-1)$th component, where $i = 1, 2, \ldots, n-(p+1)b+1$. The remaining p bursts of length b or less each will then be in the last (remaining) $(n-b-i+1)$ components. To enumerate the number of such remaining p bursts is equivalent to enumerate the number of p-repeated bursts of length b or less in a vector of length $(n-b-i+1)$. Since the result has been assumed to be true for $m = p$, so applying this to a vector of length $(n-b-i+1)$ gives the required number as

$$\begin{aligned}
&q^{p(b-1)}\left(\binom{n-b-i+1-pb+p}{p}(q-1)^p\right.\\
&\quad\left.+\sum_{\ell=0}^{p-1}\binom{n-b-i+1-pb+\ell}{\ell}(q-1)^\ell q^{p-1-\ell}\right)\\
&\quad= q^{p(b-1)}\left(\binom{n-(p+1)b-i+(p+1)}{p}(q-1)^p\right.\\
&\qquad\left.+\sum_{\ell=0}^{p-1}\binom{n-(p+1)b-i+(\ell+1)}{\ell}(q-1)^\ell q^{p-1-\ell}\right).
\end{aligned}$$

Summing on i gives the number of $(p+1)$-repeated bursts with the first non-zero component in the first $(n-(p+1)b+1)$ components as

$$\sum_{i=1}^{n-(p+1)b+1}(q-1)q^{b-1}\left[q^{p(b-1)}\left(\binom{n-(p+1)b-i+(p+1)}{p}(q-1)^p\right.\right.$$
$$\left.\left.+\sum_{l=0}^{p-1}\binom{n-(p+1)b-i+(l+1)}{l}(q-1)^l q^{p-1-l}\right)\right]$$
$$=q^{(p+1)(b-1)}\left(\sum_{i=1}^{n-(p+1)b+1}\binom{n-(p+1)b+(p+1)-i}{p}(q-1)^{p+1}\right.$$
$$\left.+\sum_{i=1}^{n-(p+1)b+1}\sum_{l=0}^{p-1}\binom{n-(p+1)b+(l+1)-i}{l}(q-1)^{l+1}q^{p-1-l}\right). \tag{3.2}$$

$$\sum_{i=1}^{n-(p+1)b+1}\binom{n-(p+1)b+(p+1)-i}{p}(q-1)^{p+1}$$
$$=\binom{n-(p+1)b+(p+1)}{p+1}(q-1)^{p+1} \qquad \text{(using (2.1))}.$$
$$\sum_{i=1}^{n-(p+1)b+1}\sum_{l=0}^{p-1}\binom{n-(p+1)b+(l+1)-i}{l}(q-1)^{l+1}q^{p-1-l}$$
$$=\sum_{l=0}^{p-1}\sum_{i=1}^{n-(p+1)b+1}\binom{n-(p+1)b+(l+1)-i}{l}(q-1)^{l+1}q^{p-1-l}$$
$$=\sum_{l=0}^{p-1}\binom{n-(p+1)b+(l+1)}{l+1}(q-1)^{l+1}q^{p-1-l} \quad \text{(using (2.1))}$$
$$=\sum_{l=1}^{p}\binom{n-(p+1)b+l}{l}(q-1)^{l}q^{p-l} \quad \text{(replacing } l+1 \text{ by } l).$$

Then (3.2) becomes

$$q^{(p+1)(b-1)}\left(\binom{n-(p+1)b+(p+1)}{p+1}(q-1)^{p+1}\right.$$
$$\left.+\sum_{l=1}^{p}\binom{n-(p+1)b+l}{l}(q-1)^{l}q^{p-l}\right).$$

Also, the number of $(p+1)$-repeated bursts in the last $(p+1)b-1$ components is clearly $q^{(p+1)b-1}$.

Thus the total number of $(p+1)$-repeated bursts of length b or less in a vector of length n is

$$\begin{aligned}
&q^{(p+1)(b-1)}\left(\binom{n-(p+1)b+(p+1)}{p+1}(q-1)^{p+1}\right.\\
&\left.+\sum_{l=1}^{p}\binom{n-(p+1)b+l}{l}(q-1)^{l}q^{p-l}\right)+q^{(p+1)b-1}\\
&=q^{(p+1)(b-1)}\left(\binom{n-(p+1)b+(p+1)}{p+1}(q-1)^{p+1}\right.\\
&\qquad\left.+\sum_{l=0}^{p}\binom{n-(p+1)b+l}{l}(q-1)^{l}q^{p-l}\right).
\end{aligned}$$

Thus the result holds for $m=(p+1)$.

Hence, by the principle of mathematical induction, the number of m-repeated bursts of length b or less in a vector of length n is

$$q^{m(b-1)}\left(\binom{n-mb+m}{m}(q-1)^{m}+\sum_{l=0}^{m-1}\binom{n-mb+l}{l}(q-1)^{l}q^{m-1-l}\right).$$

The rest of the result follows as in Theorem 2.1. □

Remark 3.1. For $m=1$, the expression obtained in (3.1) reduces to

$$q^{n-k}\geqslant q^{b-1}\left(\binom{n-b+1}{1}(q-1)+1\right)$$

which is the well-known Fire's bound (see [6]; [11, Theorem 4.16]).

Remark 3.2. For $m=2$, the expression obtained in (3.1) reduces to

$$q^{n-k}\geqslant q^{2(b-1)}\left(\binom{n-2b+2}{2}(q-1)^{2}+\sum_{l=0}^{1}\binom{n-2b+l}{l}(q-1)^{l}q^{1-l}\right)$$

which coincides with the result obtained in Theorem 2.1 for 2-repeated bursts.

In the following result we derive a sufficient condition on the number of check digits required for the existence of m-repeated burst error correcting code. The proof is based on the technique used to establish Varsharmov-Gilbert-Sacks bound by constructing a parity-check matrix for the desired code (see [11, Theorem 4.17]). This technique not only ensures the existence of such a code but also gives a method for constructing such codes.

Theorem 3.2. *There shall always exist an (n,k) linear code over $\mathrm{GF}(q)$ that cor-*

rects all m-repeated bursts of length b or less $(n > 2mb)$ *provided that*

$$q^{n-k} > q^{2m(b-1)}\left(\binom{n-2mb+(2m-1)}{2m-1}(q-1)^{2m-1} + \sum_{l=0}^{2m-2}\binom{n-2mb+l}{l}(q-1)^l q^{2m-2-l}\right). \tag{3.3}$$

Proof. We shall prove the result by constructing an appropriate $(n-k)\times n$ parity-check matrix $H = [h_1, h_2, \ldots, h_n]$ for the desired code. Choose any non-zero $(n-k)$-tuple as the first column h_1 of H. Suppose that we have selected the first $j-1$ columns $h_1, h_2, \ldots, h_{j-1}$ of H suitably. We lay down the condition to add the jth column h_j as follows:

h_j should not be a linear combination of the immediately preceding $(b-1)$ or fewer columns $h_{j-b+1}, \ldots, h_{j-1}$ of H together with any $(2m-1)$ distinct sets of b or fewer consecutive columns each from amongst the first $(j-b)$ columns $h_1, h_2, \ldots, h_{j-b}$.

In other words,

$$\begin{aligned} h_j \neq\ & (\alpha_1 h_{j-b+1} + \alpha_2 h_{j-b+2} + \ldots + \alpha_{b-1} h_{j-1}) \\ & + (\beta_1 h_{i_1} + \beta_2 h_{i_1+1} + \ldots + \beta_b h_{i_1+b-1}) \\ & + (\gamma_1 h_{i_2} + \gamma_2 h_{i_2+1} + \ldots + \gamma_b h_{i_2+b-1}) + \ldots \\ & + (\delta_1 h_{i_{2m-1}} + \delta_2 h_{i_{2m-1}+1} + \ldots + \delta_b h_{i_{2m-1}+b-1}) \end{aligned} \tag{3.4}$$

where $\alpha_i, \beta_i, \gamma_i, \ldots, \delta_i \in \mathrm{GF}(q)$ and $i_1 + i_2 + \ldots + i_{2m-1} + (2m-1)b - (2m-1) \leq j - b$. This condition ensures that there would not be a code-word which is expressible as a sum or difference of two vectors each of which is an m-repeated burst of length b or less.

We now enumerate all possible combinations on the R.H.S. of (3.4).

Clearly, the coefficients α_i's which are $(b-1)$ in number can be chosen in q^{b-1} ways.

To compute the coefficients β_i's, γ_i's, $\ldots$, δ_i's is equivalent to finding the number of $(2m-1)$-repeated bursts of length b or less in a vector of length $j-b$, which by Theorem 3.1 is

$$q^{(2m-1)(b-1)}\left(\binom{j-2mb+2m-1}{2m-1}(q-1)^{2m-1} + \sum_{l=0}^{2m-2}\binom{j-2mb+l}{l}(q-1)^l q^{2m-2-l}\right).$$

Thus, the total number of linear combinations on the R.H.S. of (3.4) equals

$$
\begin{aligned}
&q^{b-1}\Bigg[q^{(2m-1)(b-1)}\Bigg(\binom{j-2mb+2m-1}{2m-1}(q-1)^{2m-1}\\
&+\sum_{l=0}^{2m-2}\binom{j-2mb+l}{l}(q-1)^l q^{2m-2-l}\Bigg)\Bigg]\\
&\quad = q^{2m(b-1)}\Bigg(\binom{j-2mb+2m-1}{2m-1}(q-1)^{2m-1}\\
&\qquad + \sum_{l=0}^{2m-2}\binom{j-2mb+l}{l}(q-1)^l q^{2m-2-l}\Bigg). \qquad (3.5)
\end{aligned}
$$

Since these many linear combinations cannot be put to be equal to h_j and at worst all the linear combinations computed in (3.5) might yield a distinct sum, therefore in view of the fact that total number of $(n-k)$-tuples is q^{n-k}, the jth column h_j can be added to H provided that $q^{n-k} > (3.5)$.

To obtain a code of length n, replace j by n in the above inequality and we get

$$
\begin{aligned}
q^{n-k} > q^{2m(b-1)}\Bigg(&\binom{n-2mb+(2m-1)}{(2m-1)}(q-1)^{2m-1}\\
&+\sum_{l=0}^{2m-2}\binom{n-2mb+l}{l}(q-1)^l q^{2m-2-l}\Bigg). \qquad \square
\end{aligned}
$$

Remark 3.3. For $m = 1$, the expression obtained in (3.3) reduces to

$$
q^{n-k} > q^{2(b-1)}\left(\binom{n-2b+1}{1}(q-1)+1\right)
$$

which is sufficient condition of correction of (usual) bursts of length b or less (see [11, Theorem 4.17]).

Remark 3.4. For $m = 2$, the expression obtained in (3.3) reduces to

$$
\begin{aligned}
q^{n-k} > q^{4(b-1)}\Bigg(&\binom{n-4b+3}{3}(q-1)^3+\binom{n-4b+2}{2}(q-1)^2\\
&+\binom{n-4b+1}{1}(q-1)q+q^2\Bigg)
\end{aligned}
$$

which coincides with the result obtained in Theorem 2.2 for 2-repeated bursts.

4. A Sufficient Condition for the Detection of m-Repeated Bursts

In this section, we derive upper bound on the number of parity-check digits required for the detection of m-repeated burst error detecting codes. The technique followed is the same as in Theorem 2.2. This technique not only ensures the existence of such a code but also gives a method for constructing such a code.

Theorem 4.1. *There shall always exist an (n, k) linear code over* $\mathrm{GF}(q)$ *that has no m-repeated burst of length b or less as a code word provided that*

$$q^{n-k} > q^{m(b-1)}\left(\binom{n-mb+(m-1)}{(m-1)}(q-1)^{m-1} + \sum_{l=0}^{m-2}\binom{n-mb+l}{l}(q-1)^l q^{m-2-l}\right). \tag{4.1}$$

Proof. We shall prove the result by constructing an appropriate $(n-k)\times n$ parity-check matrix $H = [h_1, h_2, \ldots, h_n]$ for the desired code. Choose any non-zero $(n-k)$-tuple as the first column h_1 of H. Suppose that we have selected the first $j-1$ columns $h_1, h_2, \ldots, h_{j-1}$ of H suitably. We lay down the condition to add the jth column h_j as follows:

h_j should not be a linear combination of the immediately preceding $(b-1)$ or fewer columns $h_{j-b+1}, h_{j-b+2}, \ldots, h_{j-1}$ of H together with any $(m-1)$ distinct sets of b or fewer consecutive columns each from amongst the first $j-b$ columns $h_1, h_2, \ldots, h_{j-b}$.

In other words,

$$\begin{aligned} h_j \neq\ & (\alpha_1 h_{j-b+1} + \alpha_2 h_{j-b+2} + \ldots + \alpha_{b-1} h_{j-1}) \\ & + (\beta_1 h_{i_1} + \beta_2 h_{i_1+1} + \ldots + \beta_b h_{i_1+b-1}) \\ & + (\gamma_1 h_{i_2} + \gamma_2 h_{i_2+1} + \ldots + \gamma_b h_{i_2+b-1}) + \ldots \\ & + (\delta_1 h_{i_{m-1}} + \delta_2 h_{i_{m-1}+1} + \ldots + \delta_b h_{i_{m-1}+b-1}) \end{aligned}$$

where α_i, $\beta_i, \gamma_i, \ldots, \delta_i \in \mathrm{GF}(q)$ and

$$i_1 + i_2 + \ldots + i_{m-1} + (m-1)b - (m-1) \le j - b. \tag{4.2}$$

This condition ensures that no m-repeated burst of length b of less will be a code word.

We now enumerate all possible linear combinations on the R.H.S. of (4.2).

Clearly, the coefficients α_i's which are $(b-1)$ in number can be chosen in q^{b-1} ways.

To compute the coefficients β_i's, γ_i's$, \ldots, \delta_i$'s is equivalent to find the number of $(m-1)$-repeated bursts of length b or less in a vector of length $j-b$, which by Theorem 3.1 is

$$q^{(m-1)(b-1)}\left(\binom{j-mb+(m-1)}{(m-1)}(q-1)^{m-1} + \sum_{l=0}^{m-2}\binom{j-mb+l}{l}(q-1)^l q^{m-2-l}\right).$$

Thus, the total number of linear combinations on the R.H.S. of (4.2) equals

$$
\begin{aligned}
& q^{b-1}\Bigg[q^{(m-1)(b-1)}\left(\binom{j-mb+(m-1)}{(m-1)}(q-1)^{m-1}\right.\\
&\qquad\qquad\left.+\sum_{l=0}^{m-2}\binom{j-mb+l}{l}(q-1)^l q^{m-2-l}\right)\Bigg]\\
&= q^{m(b-1)}\left(\binom{j-mb+(m-1)}{(m-1)}(q-1)^{m-1}\right.\\
&\qquad\qquad\left.+\sum_{l=0}^{m-2}\binom{j-mb+l}{l}(q-1)^l q^{m-2-l}\right). \qquad (4.3)
\end{aligned}
$$

Since these many linear combinations cannot be put to be equal to h_j and at worst all the linear combinations computed in (4.3) might yield a distinct sum, therefore in view of the fact that the total number of $(n-k)$-tuples is q^{n-k}, the jth column h_j can be added to H provided that $q^{n-k} > (4.3)$.

To obtain a code of length n, replace j by n in the above inequality and we get

$$
\begin{aligned}
q^{n-k} > q^{m(b-1)}&\left(\binom{n-mb+(m-1)}{(m-1)}(q-1)^{m-1}\right.\\
&\left.+\sum_{l=0}^{m-2}\binom{n-mb+l}{l}(q-1)^l q^{m-2-l}\right). \qquad \square
\end{aligned}
$$

Remark 4.1. For $m=1$ the expression obtained in (4.1) reduces to

$$q^{n-k} > q^{b-1}$$

which is the sufficient condition of detection of (usual) bursts of length b or less (see [11, Theorem 4.14]).

Remark 4.2. For $m=2$, the expression obtained in (4.1) reduces to

$$q^{n-k} > q^{2(b-1)}\left(\binom{n-2b+1}{1}(q-1)+1\right)$$

which coincides with the result obtained in [2, Theorem 2] for 2-repeated bursts.

Remark 4.3. We note that the necessary condition for correction of m-repeated bursts (Theorem 3.3) is required to be proved first because it is needed to prove the sufficient condition of detection of m-repeated bursts (Theorem 4.1).

Remark 4.4. In view of the fact that the result obtained in Theorem 4.1 for the detection of $2m$-repeated bursts is the same as the result for the correction of m-repeated bursts of length b or less, it is worthwhile to note that such a code can serve dual purpose, i.e., it can either be used to correct m-repeated bursts of length b or less or it can be used to detect a $2m$-repeated burst of length b or less.

We conclude with the following two examples.

Example 4.1. Consider a $(15, 3)$ binary code with parity-check matrix

$$H = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&1&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&1 \end{bmatrix}_{12\times 15}$$

This matrix has been constructed by the synthesis procedure outlined in the proof of Theorem 2.2 by taking $b = 3$ over GF(2). It can be seen from Table 1 that the syndromes of different 2-repeated bursts of length 3 or less are distinct, showing thereby that the code that is the null space of the matrix given in the example, corrects all 2-repeated bursts of length 3 or less.

Table 1. Error Pattern – Syndrome Table.

Error vectors	Syndromes
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1 0 0 0 0 0 0 0 0 0 0 0
1 0 0 1 0 0 0 0 0 0 0 0 0 0 0	1 0 0 1 0 0 0 0 0 0 0 0
1 0 0 0 1 0 0 0 0 0 0 0 0 0 0	1 0 0 0 1 0 0 0 0 0 0 0
1 0 0 0 0 1 0 0 0 0 0 0 0 0 0	1 0 0 0 0 1 0 0 0 0 0 0
1 0 0 0 0 0 1 0 0 0 0 0 0 0 0	1 0 0 0 0 0 1 0 0 0 0 0
1 0 0 0 0 0 0 1 0 0 0 0 0 0 0	1 0 0 0 0 0 0 1 0 0 0 0
1 0 0 0 0 0 0 0 1 0 0 0 0 0 0	1 0 0 0 0 0 0 0 1 0 0 0
1 0 0 0 0 0 0 0 0 1 0 0 0 0 0	1 0 0 0 0 0 0 0 0 1 0 0
1 0 0 0 0 0 0 0 0 0 1 0 0 0 0	1 0 0 0 0 0 0 0 0 0 1 0
1 0 0 0 0 0 0 0 0 0 0 1 0 0 0	1 0 0 0 0 0 0 0 0 0 0 1
1 0 0 0 0 0 0 0 0 0 0 0 1 0 0	0 0 0 1 0 0 1 0 0 1 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 1 0	1 1 0 0 1 0 0 1 0 0 1 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 1	1 0 1 0 0 1 0 0 1 0 0 1
1 0 0 1 1 0 0 0 0 0 0 0 0 0 0	1 0 0 1 1 0 0 0 0 0 0 0
1 0 0 0 1 1 0 0 0 0 0 0 0 0 0	1 0 0 0 1 1 0 0 0 0 0 0
1 0 0 0 0 1 1 0 0 0 0 0 0 0 0	1 0 0 0 0 1 1 0 0 0 0 0
1 0 0 0 0 0 1 1 0 0 0 0 0 0 0	1 0 0 0 0 0 1 1 0 0 0 0
1 0 0 0 0 0 0 1 1 0 0 0 0 0 0	1 0 0 0 0 0 0 1 1 0 0 0
1 0 0 0 0 0 0 0 1 1 0 0 0 0 0	1 0 0 0 0 0 0 0 1 1 0 0
1 0 0 0 0 0 0 0 0 1 1 0 0 0 0	1 0 0 0 0 0 0 0 0 1 1 0
1 0 0 0 0 0 0 0 0 0 1 1 0 0 0	1 0 0 0 0 0 0 0 0 0 1 1
1 0 0 0 0 0 0 0 0 0 0 1 1 0 0	0 0 0 1 0 0 1 0 0 1 0 1
1 0 0 0 0 0 0 0 0 0 0 0 1 1 0	0 1 0 1 1 0 1 1 0 1 1 0
1 0 0 0 0 0 0 0 0 0 0 0 0 1 1	1 1 1 0 1 1 0 1 1 0 1 1
1 0 0 1 0 1 0 0 0 0 0 0 0 0 0	1 0 0 1 0 1 0 0 0 0 0 0
1 0 0 0 1 0 1 0 0 0 0 0 0 0 0	1 0 0 0 1 0 1 0 0 0 0 0
1 0 0 0 0 1 0 1 0 0 0 0 0 0 0	1 0 0 0 0 1 0 1 0 0 0 0
1 0 0 0 0 0 1 0 1 0 0 0 0 0 0	1 0 0 0 0 0 1 0 1 0 0 0
1 0 0 0 0 0 0 1 0 1 0 0 0 0 0	1 0 0 0 0 0 0 1 0 1 0 0
1 0 0 0 0 0 0 0 1 0 1 0 0 0 0	1 0 0 0 0 0 0 0 1 0 1 0
1 0 0 0 0 0 0 0 0 1 0 1 0 0 0	1 0 0 0 0 0 0 0 0 1 0 1
1 0 0 0 0 0 0 0 0 0 1 0 1 0 0	0 0 0 1 0 0 1 0 0 1 1 0
1 0 0 0 0 0 0 0 0 0 0 1 0 1 0	1 1 0 0 1 0 0 1 0 0 1 1
1 0 0 0 0 0 0 0 0 0 0 0 1 0 1	0 0 1 1 0 1 1 0 1 1 0 1
1 0 0 1 1 1 0 0 0 0 0 0 0 0 0	1 0 0 1 1 1 0 0 0 0 0 0
1 0 0 0 1 1 1 0 0 0 0 0 0 0 0	1 0 0 0 1 1 1 0 0 0 0 0

Error vectors	Syndromes
1 0 0 0 0 1 1 1 0 0 0 0 0 0 0	1 0 0 0 0 1 1 1 0 0 0 0
1 0 0 0 0 0 1 1 1 0 0 0 0 0 0	1 0 0 0 0 0 1 1 1 0 0 0
1 0 0 0 0 0 0 1 1 1 0 0 0 0 0	1 0 0 0 0 0 0 1 1 1 0 0
1 0 0 0 0 0 0 0 1 1 1 0 0 0 0	1 0 0 0 0 0 0 0 1 1 1 0
1 0 0 0 0 0 0 0 0 1 1 1 0 0 0	1 0 0 0 0 0 0 0 0 1 1 1
1 0 0 0 0 0 0 0 0 0 1 1 1 0 0	0 0 0 1 0 0 1 0 0 1 1 1
1 0 0 0 0 0 0 0 0 0 0 1 1 1 0	0 1 0 1 1 0 1 1 0 1 1 1
1 0 0 0 0 0 0 0 0 0 0 0 1 1 1	0 1 1 1 1 1 1 1 1 1 1 1
1 1 0 0 0 0 0 0 0 0 0 0 0 0 0	1 1 0 0 0 0 0 0 0 0 0 0
1 1 0 1 0 0 0 0 0 0 0 0 0 0 0	1 1 0 1 0 0 0 0 0 0 0 0
1 1 0 0 1 0 0 0 0 0 0 0 0 0 0	1 1 0 0 1 0 0 0 0 0 0 0
1 1 0 0 0 1 0 0 0 0 0 0 0 0 0	1 1 0 0 0 1 0 0 0 0 0 0
1 1 0 0 0 0 1 0 0 0 0 0 0 0 0	1 1 0 0 0 0 1 0 0 0 0 0
1 1 0 0 0 0 0 1 0 0 0 0 0 0 0	1 1 0 0 0 0 0 1 0 0 0 0
1 1 0 0 0 0 0 0 1 0 0 0 0 0 0	1 1 0 0 0 0 0 0 1 0 0 0
1 1 0 0 0 0 0 0 0 1 0 0 0 0 0	1 1 0 0 0 0 0 0 0 1 0 0
1 1 0 0 0 0 0 0 0 0 1 0 0 0 0	1 1 0 0 0 0 0 0 0 0 1 0
1 1 0 0 0 0 0 0 0 0 0 1 0 0 0	1 1 0 0 0 0 0 0 0 0 0 1
1 1 0 0 0 0 0 0 0 0 0 0 1 0 0	0 1 0 1 0 0 1 0 0 1 0 0
1 1 0 0 0 0 0 0 0 0 0 0 0 1 0	1 0 0 0 1 0 0 1 0 0 1 0
1 1 0 0 0 0 0 0 0 0 0 0 0 0 1	1 1 1 0 0 1 0 0 1 0 0 1
1 1 0 1 1 0 0 0 0 0 0 0 0 0 0	1 1 0 1 1 0 0 0 0 0 0 0
1 1 0 0 1 1 0 0 0 0 0 0 0 0 0	1 1 0 0 1 1 0 0 0 0 0 0
1 1 0 0 0 1 1 0 0 0 0 0 0 0 0	1 1 0 0 0 1 1 0 0 0 0 0
1 1 0 0 0 0 1 1 0 0 0 0 0 0 0	1 1 0 0 0 0 1 1 0 0 0 0
1 1 0 0 0 0 0 1 1 0 0 0 0 0 0	1 1 0 0 0 0 0 1 1 0 0 0
1 1 0 0 0 0 0 0 1 1 0 0 0 0 0	1 1 0 0 0 0 0 0 1 1 0 0
1 1 0 0 0 0 0 0 0 1 1 0 0 0 0	1 1 0 0 0 0 0 0 0 1 1 0
1 1 0 0 0 0 0 0 0 0 1 1 0 0 0	1 1 0 0 0 0 0 0 0 0 1 1
1 1 0 0 0 0 0 0 0 0 0 1 1 0 0	0 1 0 1 0 0 1 0 0 1 0 1
1 1 0 0 0 0 0 0 0 0 0 0 1 1 0	0 0 0 1 1 0 1 1 0 1 1 0
1 1 0 0 0 0 0 0 0 0 0 0 0 1 1	1 0 1 0 1 1 0 1 1 0 1 1
1 1 0 1 0 1 0 0 0 0 0 0 0 0 0	1 1 0 1 0 1 0 0 0 0 0 0
1 1 0 0 1 0 1 0 0 0 0 0 0 0 0	1 1 0 0 1 0 1 0 0 0 0 0
1 1 0 0 0 1 0 1 0 0 0 0 0 0 0	1 1 0 0 0 1 0 1 0 0 0 0
1 1 0 0 0 0 1 0 1 0 0 0 0 0 0	1 1 0 0 0 0 1 0 1 0 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
1 1 0 0 0 0 0 1 0 1 0 0 0 0 0	1 1 0 0 0 0 0 1 0 1 0 0
1 1 0 0 0 0 0 0 1 0 1 0 0 0 0	1 1 0 0 0 0 0 0 1 0 1 0
1 1 0 0 0 0 0 0 0 1 0 1 0 0 0	1 1 0 0 0 0 0 0 0 1 0 1
1 1 0 0 0 0 0 0 0 0 1 0 1 0 0	0 1 0 1 0 0 1 0 0 1 1 0
1 1 0 0 0 0 0 0 0 0 0 1 0 1 0	1 0 0 0 1 0 0 1 0 0 1 1
1 1 0 0 0 0 0 0 0 0 0 0 1 0 1	0 1 1 1 0 1 1 0 1 1 0 1
1 1 0 1 1 1 0 0 0 0 0 0 0 0 0	1 1 0 1 1 1 0 0 0 0 0 0
1 1 0 0 1 1 1 0 0 0 0 0 0 0 0	1 1 0 0 1 1 1 0 0 0 0 0
1 1 0 0 0 1 1 1 0 0 0 0 0 0 0	1 1 0 0 0 1 1 1 0 0 0 0
1 1 0 0 0 0 1 1 1 0 0 0 0 0 0	1 1 0 0 0 0 1 1 1 0 0 0
1 1 0 0 0 0 0 1 1 1 0 0 0 0 0	1 1 0 0 0 0 0 1 1 1 0 0
1 1 0 0 0 0 0 0 1 1 1 0 0 0 0	1 1 0 0 0 0 0 0 1 1 1 0
1 1 0 0 0 0 0 0 0 1 1 1 0 0 0	1 1 0 0 0 0 0 0 0 1 1 1
1 1 0 0 0 0 0 0 0 0 1 1 1 0 0	0 1 0 1 0 0 1 0 0 1 1 1
1 1 0 0 0 0 0 0 0 0 0 1 1 1 0	0 0 0 1 1 0 1 1 0 1 1 1
1 1 0 0 0 0 0 0 0 0 0 0 1 1 1	0 0 1 1 1 1 1 1 1 1 1 1
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0	1 0 1 0 0 0 0 0 0 0 0 0
1 0 1 1 0 0 0 0 0 0 0 0 0 0 0	1 0 1 1 0 0 0 0 0 0 0 0
1 0 1 0 1 0 0 0 0 0 0 0 0 0 0	1 0 1 0 1 0 0 0 0 0 0 0
1 0 1 0 0 1 0 0 0 0 0 0 0 0 0	1 0 1 0 0 1 0 0 0 0 0 0
1 0 1 0 0 0 1 0 0 0 0 0 0 0 0	1 0 1 0 0 0 1 0 0 0 0 0
1 0 1 0 0 0 0 1 0 0 0 0 0 0 0	1 0 1 0 0 0 0 1 0 0 0 0
1 0 1 0 0 0 0 0 1 0 0 0 0 0 0	1 0 1 0 0 0 0 0 1 0 0 0
1 0 1 0 0 0 0 0 0 1 0 0 0 0 0	1 0 1 0 0 0 0 0 0 1 0 0
1 0 1 0 0 0 0 0 0 0 1 0 0 0 0	1 0 1 0 0 0 0 0 0 0 1 0
1 0 1 0 0 0 0 0 0 0 0 1 0 0 0	1 0 1 0 0 0 0 0 0 0 0 1
1 0 1 0 0 0 0 0 0 0 0 0 1 0 0	0 0 1 1 0 0 1 0 0 1 0 0
1 0 1 0 0 0 0 0 0 0 0 0 0 1 0	1 1 1 0 1 0 0 1 0 0 1 0
1 0 1 0 0 0 0 0 0 0 0 0 0 0 1	1 0 0 0 0 1 0 0 1 0 0 1
1 0 1 1 1 0 0 0 0 0 0 0 0 0 0	1 0 1 1 1 0 0 0 0 0 0 0
1 0 1 0 1 1 0 0 0 0 0 0 0 0 0	1 0 1 0 1 1 0 0 0 0 0 0
1 0 1 0 0 1 1 0 0 0 0 0 0 0 0	1 0 1 0 0 1 1 0 0 0 0 0
1 0 1 0 0 0 1 1 0 0 0 0 0 0 0	1 0 1 0 0 0 1 1 0 0 0 0
1 0 1 0 0 0 0 1 1 0 0 0 0 0 0	1 0 1 0 0 0 0 1 1 0 0 0
1 0 1 0 0 0 0 0 1 1 0 0 0 0 0	1 0 1 0 0 0 0 0 1 1 0 0
1 0 1 0 0 0 0 0 0 1 1 0 0 0 0	1 0 1 0 0 0 0 0 0 1 1 0
1 0 1 0 0 0 0 0 0 0 1 1 0 0 0	1 0 1 0 0 0 0 0 0 0 1 1
1 0 1 0 0 0 0 0 0 0 0 1 1 0 0	0 0 1 1 0 0 1 0 0 1 0 1
1 0 1 0 0 0 0 0 0 0 0 0 1 1 0	0 1 1 1 1 0 1 1 0 1 1 0

Error vectors	Syndromes
1 0 1 0 0 0 0 0 0 0 0 0 0 1 1	1 1 0 0 1 1 0 1 1 0 1 1
1 0 1 1 0 1 0 0 0 0 0 0 0 0 0	1 0 1 1 0 1 0 0 0 0 0 0
1 0 1 0 1 0 1 0 0 0 0 0 0 0 0	1 0 1 0 1 0 1 0 0 0 0 0
1 0 1 0 0 1 0 1 0 0 0 0 0 0 0	1 0 1 0 0 1 0 1 0 0 0 0
1 0 1 0 0 0 1 0 1 0 0 0 0 0 0	1 0 1 0 0 0 1 0 1 0 0 0
1 0 1 0 0 0 0 1 0 1 0 0 0 0 0	1 0 1 0 0 0 0 1 0 1 0 0
1 0 1 0 0 0 0 0 1 0 1 0 0 0 0	1 0 1 0 0 0 0 0 1 0 1 0
1 0 1 0 0 0 0 0 0 1 0 1 0 0 0	1 0 1 0 0 0 0 0 0 1 0 1
1 0 1 0 0 0 0 0 0 0 1 0 1 0 0	0 0 1 1 0 0 1 0 0 1 1 0
1 0 1 0 0 0 0 0 0 0 0 1 0 1 0	1 1 1 0 1 0 0 1 0 0 1 1
1 0 1 0 0 0 0 0 0 0 0 0 1 0 1	0 0 0 1 0 1 1 0 1 1 0 1
1 0 1 1 1 1 0 0 0 0 0 0 0 0 0	1 0 1 1 1 1 0 0 0 0 0 0
1 0 1 0 1 1 1 0 0 0 0 0 0 0 0	1 0 1 0 1 1 1 0 0 0 0 0
1 0 1 0 0 1 1 1 0 0 0 0 0 0 0	1 0 1 0 0 1 1 1 0 0 0 0
1 0 1 0 0 0 1 1 1 0 0 0 0 0 0	1 0 1 0 0 0 1 1 1 0 0 0
1 0 1 0 0 0 0 1 1 1 0 0 0 0 0	1 0 1 0 0 0 0 1 1 1 0 0
1 0 1 0 0 0 0 0 1 1 1 0 0 0 0	1 0 1 0 0 0 0 0 1 1 1 0
1 0 1 0 0 0 0 0 0 1 1 1 0 0 0	1 0 1 0 0 0 0 0 0 1 1 1
1 0 1 0 0 0 0 0 0 0 1 1 1 0 0	0 0 1 1 0 0 1 0 0 1 1 1
1 0 1 0 0 0 0 0 0 0 0 1 1 1 0	0 1 1 1 1 0 1 1 0 1 1 1
1 0 1 0 0 0 0 0 0 0 0 0 1 1 1	0 1 0 1 1 1 1 1 1 1 1 1
1 1 1 0 0 0 0 0 0 0 0 0 0 0 0	1 1 1 0 0 0 0 0 0 0 0 0
1 1 1 1 0 0 0 0 0 0 0 0 0 0 0	1 1 1 1 0 0 0 0 0 0 0 0
1 1 1 0 1 0 0 0 0 0 0 0 0 0 0	1 1 1 0 1 0 0 0 0 0 0 0
1 1 1 0 0 1 0 0 0 0 0 0 0 0 0	1 1 1 0 0 1 0 0 0 0 0 0
1 1 1 0 0 0 1 0 0 0 0 0 0 0 0	1 1 1 0 0 0 1 0 0 0 0 0
1 1 1 0 0 0 0 1 0 0 0 0 0 0 0	1 1 1 0 0 0 0 1 0 0 0 0
1 1 1 0 0 0 0 0 1 0 0 0 0 0 0	1 1 1 0 0 0 0 0 1 0 0 0
1 1 1 0 0 0 0 0 0 1 0 0 0 0 0	1 1 1 0 0 0 0 0 0 1 0 0
1 1 1 0 0 0 0 0 0 0 1 0 0 0 0	1 1 1 0 0 0 0 0 0 0 1 0
1 1 1 0 0 0 0 0 0 0 0 1 0 0 0	1 1 1 0 0 0 0 0 0 0 0 1
1 1 1 0 0 0 0 0 0 0 0 0 1 0 0	0 1 1 1 0 0 1 0 0 1 0 0
1 1 1 0 0 0 0 0 0 0 0 0 0 1 0	1 0 1 0 1 0 0 1 0 0 1 0
1 1 1 0 0 0 0 0 0 0 0 0 0 0 1	1 1 0 0 0 1 0 0 1 0 0 1
1 1 1 1 1 0 0 0 0 0 0 0 0 0 0	1 1 1 1 1 0 0 0 0 0 0 0
1 1 1 0 1 1 0 0 0 0 0 0 0 0 0	1 1 1 0 1 1 0 0 0 0 0 0
1 1 1 0 0 1 1 0 0 0 0 0 0 0 0	1 1 1 0 0 1 1 0 0 0 0 0
1 1 1 0 0 0 1 1 0 0 0 0 0 0 0	1 1 1 0 0 0 1 1 0 0 0 0
1 1 1 0 0 0 0 1 1 0 0 0 0 0 0	1 1 1 0 0 0 0 1 1 0 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
1 1 1 0 0 0 0 0 1 1 0 0 0 0 0	1 1 1 0 0 0 0 0 1 1 0 0
1 1 1 0 0 0 0 0 0 1 1 0 0 0 0	1 1 1 0 0 0 0 0 0 1 1 0
1 1 1 0 0 0 0 0 0 0 1 1 0 0 0	1 1 1 0 0 0 0 0 0 0 1 1
1 1 1 0 0 0 0 0 0 0 0 1 1 0 0	0 1 1 1 0 0 1 0 0 1 0 1
1 1 1 0 0 0 0 0 0 0 0 0 1 1 0	0 0 1 1 1 0 1 1 0 1 1 0
1 1 1 0 0 0 0 0 0 0 0 0 0 1 1	1 0 0 0 1 1 0 1 1 0 1 1
1 1 1 1 0 1 0 0 0 0 0 0 0 0 0	1 1 1 1 0 1 0 0 0 0 0 0
1 1 1 0 1 0 1 0 0 0 0 0 0 0 0	1 1 1 0 1 0 1 0 0 0 0 0
1 1 1 0 0 1 0 1 0 0 0 0 0 0 0	1 1 1 0 0 1 0 1 0 0 0 0
1 1 1 0 0 0 1 0 1 0 0 0 0 0 0	1 1 1 0 0 0 1 0 1 0 0 0
1 1 1 0 0 0 0 1 0 1 0 0 0 0 0	1 1 1 0 0 0 0 1 0 1 0 0
1 1 1 0 0 0 0 0 1 0 1 0 0 0 0	1 1 1 0 0 0 0 0 1 0 1 0
1 1 1 0 0 0 0 0 0 1 0 1 0 0 0	1 1 1 0 0 0 0 0 0 1 0 1
1 1 1 0 0 0 0 0 0 0 1 0 1 0 0	0 1 1 1 0 0 1 0 0 1 1 0
1 1 1 0 0 0 0 0 0 0 0 1 0 1 0	1 0 1 0 1 0 0 1 0 0 1 1
1 1 1 0 0 0 0 0 0 0 0 0 1 0 1	0 1 0 1 0 1 1 0 1 1 0 1
1 1 1 1 1 1 0 0 0 0 0 0 0 0 0	1 1 1 1 1 1 0 0 0 0 0 0
1 1 1 0 1 1 1 0 0 0 0 0 0 0 0	1 1 1 0 1 1 1 0 0 0 0 0
1 1 1 0 0 1 1 1 0 0 0 0 0 0 0	1 1 1 0 0 1 1 1 0 0 0 0
1 1 1 0 0 0 1 1 1 0 0 0 0 0 0	1 1 1 0 0 0 1 1 1 0 0 0
1 1 1 0 0 0 0 1 1 1 0 0 0 0 0	1 1 1 0 0 0 0 1 1 1 0 0
1 1 1 0 0 0 0 0 1 1 1 0 0 0 0	1 1 1 0 0 0 0 0 1 1 1 0
1 1 1 0 0 0 0 0 0 1 1 1 0 0 0	1 1 1 0 0 0 0 0 0 1 1 1
1 1 1 0 0 0 0 0 0 0 1 1 1 0 0	0 1 1 1 0 0 1 0 0 1 1 1
1 1 1 0 0 0 0 0 0 0 0 1 1 1 0	0 0 1 1 1 0 1 1 0 1 1 1
1 1 1 0 0 0 0 0 0 0 0 0 1 1 1	0 0 0 1 1 1 1 1 1 1 1 1
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0	0 1 0 0 0 0 0 0 0 0 0 0
0 1 0 0 1 0 0 0 0 0 0 0 0 0 0	0 1 0 0 1 0 0 0 0 0 0 0
0 1 0 0 0 1 0 0 0 0 0 0 0 0 0	0 1 0 0 0 1 0 0 0 0 0 0
0 1 0 0 0 0 1 0 0 0 0 0 0 0 0	0 1 0 0 0 0 1 0 0 0 0 0
0 1 0 0 0 0 0 1 0 0 0 0 0 0 0	0 1 0 0 0 0 0 1 0 0 0 0
0 1 0 0 0 0 0 0 1 0 0 0 0 0 0	0 1 0 0 0 0 0 0 1 0 0 0
0 1 0 0 0 0 0 0 0 1 0 0 0 0 0	0 1 0 0 0 0 0 0 0 1 0 0
0 1 0 0 0 0 0 0 0 0 1 0 0 0 0	0 1 0 0 0 0 0 0 0 0 1 0
0 1 0 0 0 0 0 0 0 0 0 1 0 0 0	0 1 0 0 0 0 0 0 0 0 0 1
0 1 0 0 0 0 0 0 0 0 0 0 1 0 0	1 1 0 1 0 0 1 0 0 1 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 1 0	0 0 0 0 1 0 0 1 0 0 1 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 1	0 1 1 0 0 1 0 0 1 0 0 1
0 1 0 0 1 1 0 0 0 0 0 0 0 0 0	0 1 0 0 1 1 0 0 0 0 0 0

Error vectors	Syndromes
0 1 0 0 0 1 1 0 0 0 0 0 0 0 0	0 1 0 0 0 1 1 0 0 0 0 0
0 1 0 0 0 0 1 1 0 0 0 0 0 0 0	0 1 0 0 0 0 1 1 0 0 0 0
0 1 0 0 0 0 0 1 1 0 0 0 0 0 0	0 1 0 0 0 0 0 1 1 0 0 0
0 1 0 0 0 0 0 0 1 1 0 0 0 0 0	0 1 0 0 0 0 0 0 1 1 0 0
0 1 0 0 0 0 0 0 0 1 1 0 0 0 0	0 1 0 0 0 0 0 0 0 1 1 0
0 1 0 0 0 0 0 0 0 0 1 1 0 0 0	0 1 0 0 0 0 0 0 0 0 1 1
0 1 0 0 0 0 0 0 0 0 0 1 1 0 0	1 1 0 1 0 0 1 0 0 1 0 1
0 1 0 0 0 0 0 0 0 0 0 0 1 1 0	1 0 0 1 1 0 1 1 0 1 1 0
0 1 0 0 0 0 0 0 0 0 0 0 0 1 1	0 0 1 0 1 1 0 1 1 0 1 1
0 1 0 0 1 0 1 0 0 0 0 0 0 0 0	0 1 0 0 1 0 1 0 0 0 0 0
0 1 0 0 0 1 0 1 0 0 0 0 0 0 0	0 1 0 0 0 1 0 1 0 0 0 0
0 1 0 0 0 0 1 0 1 0 0 0 0 0 0	0 1 0 0 0 0 1 0 1 0 0 0
0 1 0 0 0 0 0 1 0 1 0 0 0 0 0	0 1 0 0 0 0 0 1 0 1 0 0
0 1 0 0 0 0 0 0 1 0 1 0 0 0 0	0 1 0 0 0 0 0 0 1 0 1 0
0 1 0 0 0 0 0 0 0 1 0 1 0 0 0	0 1 0 0 0 0 0 0 0 1 0 1
0 1 0 0 0 0 0 0 0 0 1 0 1 0 0	1 1 0 1 0 0 1 0 0 1 1 0
0 1 0 0 0 0 0 0 0 0 0 1 0 1 0	0 0 0 0 1 0 0 1 0 0 1 1
0 1 0 0 0 0 0 0 0 0 0 0 1 0 1	1 1 1 1 0 1 1 0 1 1 0 1
0 1 0 0 1 1 1 0 0 0 0 0 0 0 0	0 1 0 0 1 1 1 0 0 0 0 0
0 1 0 0 0 1 1 1 0 0 0 0 0 0 0	0 1 0 0 0 1 1 1 0 0 0 0
0 1 0 0 0 0 1 1 1 0 0 0 0 0 0	0 1 0 0 0 0 1 1 1 0 0 0
0 1 0 0 0 0 0 1 1 1 0 0 0 0 0	0 1 0 0 0 0 0 1 1 1 0 0
0 1 0 0 0 0 0 0 1 1 1 0 0 0 0	0 1 0 0 0 0 0 0 1 1 1 0
0 1 0 0 0 0 0 0 0 1 1 1 0 0 0	0 1 0 0 0 0 0 0 0 1 1 1
0 1 0 0 0 0 0 0 0 0 1 1 1 0 0	1 1 0 1 0 0 1 0 0 1 1 1
0 1 0 0 0 0 0 0 0 0 0 1 1 1 0	1 0 0 1 1 0 1 1 0 1 1 1
0 1 0 0 0 0 0 0 0 0 0 0 1 1 1	1 0 1 1 1 1 1 1 1 1 1 1
0 1 1 0 0 0 0 0 0 0 0 0 0 0 0	0 1 1 0 0 0 0 0 0 0 0 0
0 1 1 0 1 0 0 0 0 0 0 0 0 0 0	0 1 1 0 1 0 0 0 0 0 0 0
0 1 1 0 0 1 0 0 0 0 0 0 0 0 0	0 1 1 0 0 1 0 0 0 0 0 0
0 1 1 0 0 0 1 0 0 0 0 0 0 0 0	0 1 1 0 0 0 1 0 0 0 0 0
0 1 1 0 0 0 0 1 0 0 0 0 0 0 0	0 1 1 0 0 0 0 1 0 0 0 0
0 1 1 0 0 0 0 0 1 0 0 0 0 0 0	0 1 1 0 0 0 0 0 1 0 0 0
0 1 1 0 0 0 0 0 0 1 0 0 0 0 0	0 1 1 0 0 0 0 0 0 1 0 0
0 1 1 0 0 0 0 0 0 0 1 0 0 0 0	0 1 1 0 0 0 0 0 0 0 1 0
0 1 1 0 0 0 0 0 0 0 0 1 0 0 0	0 1 1 0 0 0 0 0 0 0 0 1
0 1 1 0 0 0 0 0 0 0 0 0 1 0 0	1 1 1 1 0 0 1 0 0 1 0 0
0 1 1 0 0 0 0 0 0 0 0 0 0 1 0	0 0 1 0 1 0 0 1 0 0 1 0
0 1 1 0 0 0 0 0 0 0 0 0 0 0 1	0 1 0 0 0 1 0 0 1 0 0 1

Table 1 (*Continued*)

Error vectors	Syndromes
0 1 1 0 1 1 0 0 0 0 0 0 0 0 0	0 1 1 0 1 1 0 0 0 0 0 0
0 1 1 0 0 1 1 0 0 0 0 0 0 0 0	0 1 1 0 0 1 1 0 0 0 0 0
0 1 1 0 0 0 1 1 0 0 0 0 0 0 0	0 1 1 0 0 0 1 1 0 0 0 0
0 1 1 0 0 0 0 1 1 0 0 0 0 0 0	0 1 1 0 0 0 0 1 1 0 0 0
0 1 1 0 0 0 0 0 1 1 0 0 0 0 0	0 1 1 0 0 0 0 0 1 1 0 0
0 1 1 0 0 0 0 0 0 1 1 0 0 0 0	0 1 1 0 0 0 0 0 0 1 1 0
0 1 1 0 0 0 0 0 0 0 1 1 0 0 0	0 1 1 0 0 0 0 0 0 0 1 1
0 1 1 0 0 0 0 0 0 0 0 1 1 0 0	1 1 1 1 0 0 1 0 0 1 0 1
0 1 1 0 0 0 0 0 0 0 0 0 1 1 0	1 0 1 1 1 0 1 1 0 1 1 0
0 1 1 0 0 0 0 0 0 0 0 0 0 1 1	0 0 0 0 1 1 0 1 1 0 1 1
0 1 1 0 1 0 1 0 0 0 0 0 0 0 0	0 1 1 0 1 0 1 0 0 0 0 0
0 1 1 0 0 1 0 1 0 0 0 0 0 0 0	0 1 1 0 0 1 0 1 0 0 0 0
0 1 1 0 0 0 1 0 1 0 0 0 0 0 0	0 1 1 0 0 0 1 0 1 0 0 0
0 1 1 0 0 0 0 1 0 1 0 0 0 0 0	0 1 1 0 0 0 0 1 0 1 0 0
0 1 1 0 0 0 0 0 1 0 1 0 0 0 0	0 1 1 0 0 0 0 0 1 0 1 0
0 1 1 0 0 0 0 0 0 1 0 1 0 0 0	0 1 1 0 0 0 0 0 0 1 0 1
0 1 1 0 0 0 0 0 0 0 1 0 1 0 0	1 1 1 1 0 0 1 0 0 1 1 0
0 1 1 0 0 0 0 0 0 0 0 1 0 1 0	0 0 1 0 1 0 0 1 0 0 1 1
0 1 1 0 0 0 0 0 0 0 0 0 1 0 1	1 1 0 1 0 1 1 0 1 1 0 1
0 1 1 0 1 1 1 0 0 0 0 0 0 0 0	0 1 1 0 1 1 1 0 0 0 0 0
0 1 1 0 0 1 1 1 0 0 0 0 0 0 0	0 1 1 0 0 1 1 1 0 0 0 0
0 1 1 0 0 0 1 1 1 0 0 0 0 0 0	0 1 1 0 0 0 1 1 1 0 0 0
0 1 1 0 0 0 0 1 1 1 0 0 0 0 0	0 1 1 0 0 0 0 1 1 1 0 0
0 1 1 0 0 0 0 0 1 1 1 0 0 0 0	0 1 1 0 0 0 0 0 1 1 1 0
0 1 1 0 0 0 0 0 0 1 1 1 0 0 0	0 1 1 0 0 0 0 0 0 1 1 1
0 1 1 0 0 0 0 0 0 0 1 1 1 0 0	1 1 1 1 0 0 1 0 0 1 1 1
0 1 1 0 0 0 0 0 0 0 0 1 1 1 0	1 0 1 1 1 0 1 1 0 1 1 1
0 1 1 0 0 0 0 0 0 0 0 0 1 1 1	1 0 0 1 1 1 1 1 1 1 1 1
0 1 0 1 0 0 0 0 0 0 0 0 0 0 0	0 1 0 1 0 0 0 0 0 0 0 0
0 1 0 1 1 0 0 0 0 0 0 0 0 0 0	0 1 0 1 1 0 0 0 0 0 0 0
0 1 0 1 0 1 0 0 0 0 0 0 0 0 0	0 1 0 1 0 1 0 0 0 0 0 0
0 1 0 1 0 0 1 0 0 0 0 0 0 0 0	0 1 0 1 0 0 1 0 0 0 0 0
0 1 0 1 0 0 0 1 0 0 0 0 0 0 0	0 1 0 1 0 0 0 1 0 0 0 0
0 1 0 1 0 0 0 0 1 0 0 0 0 0 0	0 1 0 1 0 0 0 0 1 0 0 0
0 1 0 1 0 0 0 0 0 1 0 0 0 0 0	0 1 0 1 0 0 0 0 0 1 0 0
0 1 0 1 0 0 0 0 0 0 1 0 0 0 0	0 1 0 1 0 0 0 0 0 0 1 0
0 1 0 1 0 0 0 0 0 0 0 1 0 0 0	0 1 0 1 0 0 0 0 0 0 0 1
0 1 0 1 0 0 0 0 0 0 0 0 1 0 0	1 1 0 0 0 0 1 0 0 1 0 0
0 1 0 1 0 0 0 0 0 0 0 0 0 1 0	0 0 0 1 1 0 0 1 0 0 1 0

Error vectors	Syndromes
0 1 0 1 0 0 0 0 0 0 0 0 0 0 1	0 1 1 1 0 1 0 0 1 0 0 1
0 1 0 1 1 1 0 0 0 0 0 0 0 0 0	0 1 0 1 1 1 0 0 0 0 0 0
0 1 0 1 0 1 1 0 0 0 0 0 0 0 0	0 1 0 1 0 1 1 0 0 0 0 0
0 1 0 1 0 0 1 1 0 0 0 0 0 0 0	0 1 0 1 0 0 1 1 0 0 0 0
0 1 0 1 0 0 0 1 1 0 0 0 0 0 0	0 1 0 1 0 0 0 1 1 0 0 0
0 1 0 1 0 0 0 0 1 1 0 0 0 0 0	0 1 0 1 0 0 0 0 1 1 0 0
0 1 0 1 0 0 0 0 0 1 1 0 0 0 0	0 1 0 1 0 0 0 0 0 1 1 0
0 1 0 1 0 0 0 0 0 0 1 1 0 0 0	0 1 0 1 0 0 0 0 0 0 1 1
0 1 0 1 0 0 0 0 0 0 0 1 1 0 0	1 1 0 0 0 0 1 0 0 1 0 1
0 1 0 1 0 0 0 0 0 0 0 0 1 1 0	1 0 0 0 1 0 1 1 0 1 1 0
0 1 0 1 0 0 0 0 0 0 0 0 0 1 1	0 0 1 1 1 1 0 1 1 0 1 1
0 1 0 1 1 0 1 0 0 0 0 0 0 0 0	0 1 0 1 1 0 1 0 0 0 0 0
0 1 0 1 0 1 0 1 0 0 0 0 0 0 0	0 1 0 1 0 1 0 1 0 0 0 0
0 1 0 1 0 0 1 0 1 0 0 0 0 0 0	0 1 0 1 0 0 1 0 1 0 0 0
0 1 0 1 0 0 0 1 0 1 0 0 0 0 0	0 1 0 1 0 0 0 1 0 1 0 0
0 1 0 1 0 0 0 0 1 0 1 0 0 0 0	0 1 0 1 0 0 0 0 1 0 1 0
0 1 0 1 0 0 0 0 0 1 0 1 0 0 0	0 1 0 1 0 0 0 0 0 1 0 1
0 1 0 1 0 0 0 0 0 0 1 0 1 0 0	1 1 0 0 0 0 1 0 0 1 1 0
0 1 0 1 0 0 0 0 0 0 0 1 0 1 0	0 0 0 1 1 0 0 1 0 0 1 1
0 1 0 1 0 0 0 0 0 0 0 0 1 0 1	1 1 1 0 0 1 1 0 1 1 0 1
0 1 0 1 1 1 1 0 0 0 0 0 0 0 0	0 1 0 1 1 1 1 0 0 0 0 0
0 1 0 1 0 1 1 1 0 0 0 0 0 0 0	0 1 0 1 0 1 1 1 0 0 0 0
0 1 0 1 0 0 1 1 1 0 0 0 0 0 0	0 1 0 1 0 0 1 1 1 0 0 0
0 1 0 1 0 0 0 1 1 1 0 0 0 0 0	0 1 0 1 0 0 0 1 1 1 0 0
0 1 0 1 0 0 0 0 1 1 1 0 0 0 0	0 1 0 1 0 0 0 0 1 1 1 0
0 1 0 1 0 0 0 0 0 1 1 1 0 0 0	0 1 0 1 0 0 0 0 0 1 1 1
0 1 0 1 0 0 0 0 0 0 1 1 1 0 0	1 1 0 0 0 0 1 0 0 1 1 1
0 1 0 1 0 0 0 0 0 0 0 1 1 1 0	1 0 0 0 1 0 1 1 0 1 1 1
0 1 0 1 0 0 0 0 0 0 0 0 1 1 1	1 0 1 0 1 1 1 1 1 1 1 1
0 1 1 1 0 0 0 0 0 0 0 0 0 0 0	0 1 1 1 0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 0 0 0 0 0 0 0 0	0 1 1 1 1 0 0 0 0 0 0 0
0 1 1 1 0 1 0 0 0 0 0 0 0 0 0	0 1 1 1 0 1 0 0 0 0 0 0
0 1 1 1 0 0 1 0 0 0 0 0 0 0 0	0 1 1 1 0 0 1 0 0 0 0 0
0 1 1 1 0 0 0 1 0 0 0 0 0 0 0	0 1 1 1 0 0 0 1 0 0 0 0
0 1 1 1 0 0 0 0 1 0 0 0 0 0 0	0 1 1 1 0 0 0 0 1 0 0 0
0 1 1 1 0 0 0 0 0 1 0 0 0 0 0	0 1 1 1 0 0 0 0 0 1 0 0
0 1 1 1 0 0 0 0 0 0 1 0 0 0 0	0 1 1 1 0 0 0 0 0 0 1 0
0 1 1 1 0 0 0 0 0 0 0 1 0 0 0	0 1 1 1 0 0 0 0 0 0 0 1
0 1 1 1 0 0 0 0 0 0 0 0 1 0 0	1 1 1 0 0 0 1 0 0 1 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 1 1 1 0 0 0 0 0 0 0 0 0 1 0	0 0 1 1 1 0 0 1 0 0 1 0
0 1 1 1 0 0 0 0 0 0 0 0 0 0 1	0 1 0 1 0 1 0 0 1 0 0 1
0 1 1 1 1 1 0 0 0 0 0 0 0 0 0	0 1 1 1 1 1 0 0 0 0 0 0
0 1 1 1 0 1 1 0 0 0 0 0 0 0 0	0 1 1 1 0 1 1 0 0 0 0 0
0 1 1 1 0 0 1 1 0 0 0 0 0 0 0	0 1 1 1 0 0 1 1 0 0 0 0
0 1 1 1 0 0 0 1 1 0 0 0 0 0 0	0 1 1 1 0 0 0 1 1 0 0 0
0 1 1 1 0 0 0 0 1 1 0 0 0 0 0	0 1 1 1 0 0 0 0 1 1 0 0
0 1 1 1 0 0 0 0 0 1 1 0 0 0 0	0 1 1 1 0 0 0 0 0 1 1 0
0 1 1 1 0 0 0 0 0 0 1 1 0 0 0	0 1 1 1 0 0 0 0 0 0 1 1
0 1 1 1 0 0 0 0 0 0 0 1 1 0 0	1 1 1 0 0 0 1 0 0 1 0 1
0 1 1 1 0 0 0 0 0 0 0 0 1 1 0	1 0 1 0 1 0 1 1 0 1 1 0
0 1 1 1 0 0 0 0 0 0 0 0 0 1 1	0 0 0 1 1 1 0 1 1 0 1 1
0 1 1 1 1 0 1 0 0 0 0 0 0 0 0	0 1 1 1 1 0 1 0 0 0 0 0
0 1 1 1 0 1 0 1 0 0 0 0 0 0 0	0 1 1 1 0 1 0 1 0 0 0 0
0 1 1 1 0 0 1 0 1 0 0 0 0 0 0	0 1 1 1 0 0 1 0 1 0 0 0
0 1 1 1 0 0 0 1 0 1 0 0 0 0 0	0 1 1 1 0 0 0 1 0 1 0 0
0 1 1 1 0 0 0 0 1 0 1 0 0 0 0	0 1 1 1 0 0 0 0 1 0 1 0
0 1 1 1 0 0 0 0 0 1 0 1 0 0 0	0 1 1 1 0 0 0 0 0 1 0 1
0 1 1 1 0 0 0 0 0 0 1 0 1 0 0	1 1 1 0 0 0 1 0 0 1 1 0
0 1 1 1 0 0 0 0 0 0 0 1 0 1 0	0 0 1 1 1 0 0 1 0 0 1 1
0 1 1 1 0 0 0 0 0 0 0 0 1 0 1	1 1 0 0 0 1 1 0 1 1 0 1
0 1 1 1 1 1 1 0 0 0 0 0 0 0 0	0 1 1 1 1 1 1 0 0 0 0 0
0 1 1 1 0 1 1 1 0 0 0 0 0 0 0	0 1 1 1 0 1 1 1 0 0 0 0
0 1 1 1 0 0 1 1 1 0 0 0 0 0 0	0 1 1 1 0 0 1 1 1 0 0 0
0 1 1 1 0 0 0 1 1 1 0 0 0 0 0	0 1 1 1 0 0 0 1 1 1 0 0
0 1 1 1 0 0 0 0 1 1 1 0 0 0 0	0 1 1 1 0 0 0 0 1 1 1 0
0 1 1 1 0 0 0 0 0 1 1 1 0 0 0	0 1 1 1 0 0 0 0 0 1 1 1
0 1 1 1 0 0 0 0 0 0 1 1 1 0 0	1 1 1 0 0 0 1 0 0 1 1 1
0 1 1 1 0 0 0 0 0 0 0 1 1 1 0	1 0 1 0 1 0 1 1 0 1 1 1
0 1 1 1 0 0 0 0 0 0 0 0 1 1 1	1 0 0 0 1 1 1 1 1 1 1 1
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0	0 0 1 0 0 0 0 0 0 0 0 0
0 0 1 0 0 1 0 0 0 0 0 0 0 0 0	0 0 1 0 0 1 0 0 0 0 0 0
0 0 1 0 0 0 1 0 0 0 0 0 0 0 0	0 0 1 0 0 0 1 0 0 0 0 0
0 0 1 0 0 0 0 1 0 0 0 0 0 0 0	0 0 1 0 0 0 0 1 0 0 0 0
0 0 1 0 0 0 0 0 1 0 0 0 0 0 0	0 0 1 0 0 0 0 0 1 0 0 0
0 0 1 0 0 0 0 0 0 1 0 0 0 0 0	0 0 1 0 0 0 0 0 0 1 0 0
0 0 1 0 0 0 0 0 0 0 1 0 0 0 0	0 0 1 0 0 0 0 0 0 0 1 0
0 0 1 0 0 0 0 0 0 0 0 1 0 0 0	0 0 1 0 0 0 0 0 0 0 0 1
0 0 1 0 0 0 0 0 0 0 0 0 1 0 0	1 0 1 1 0 0 1 0 0 1 0 0

Error vectors	Syndromes
0 0 1 0 0 0 0 0 0 0 0 0 0 1 0	0 1 1 0 1 0 0 1 0 0 1 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 1	0 0 0 0 0 1 0 0 1 0 0 1
0 0 1 0 0 1 1 0 0 0 0 0 0 0 0	0 0 1 0 0 1 1 0 0 0 0 0
0 0 1 0 0 0 1 1 0 0 0 0 0 0 0	0 0 1 0 0 0 1 1 0 0 0 0
0 0 1 0 0 0 0 1 1 0 0 0 0 0 0	0 0 1 0 0 0 0 1 1 0 0 0
0 0 1 0 0 0 0 0 1 1 0 0 0 0 0	0 0 1 0 0 0 0 0 1 1 0 0
0 0 1 0 0 0 0 0 0 1 1 0 0 0 0	0 0 1 0 0 0 0 0 0 1 1 0
0 0 1 0 0 0 0 0 0 0 1 1 0 0 0	0 0 1 0 0 0 0 0 0 0 1 1
0 0 1 0 0 0 0 0 0 0 0 1 1 0 0	1 0 1 1 0 0 1 0 0 1 0 1
0 0 1 0 0 0 0 0 0 0 0 0 1 1 0	1 1 1 1 1 0 1 1 0 1 1 0
0 0 1 0 0 0 0 0 0 0 0 0 0 1 1	0 1 0 0 1 1 0 1 1 0 1 1
0 0 1 0 0 1 0 1 0 0 0 0 0 0 0	0 0 1 0 0 1 0 1 0 0 0 0
0 0 1 0 0 0 1 0 1 0 0 0 0 0 0	0 0 1 0 0 0 1 0 1 0 0 0
0 0 1 0 0 0 0 1 0 1 0 0 0 0 0	0 0 1 0 0 0 0 1 0 1 0 0
0 0 1 0 0 0 0 0 1 0 1 0 0 0 0	0 0 1 0 0 0 0 0 1 0 1 0
0 0 1 0 0 0 0 0 0 1 0 1 0 0 0	0 0 1 0 0 0 0 0 0 1 0 1
0 0 1 0 0 0 0 0 0 0 1 0 1 0 0	1 0 1 1 0 0 1 0 0 1 1 0
0 0 1 0 0 0 0 0 0 0 0 1 0 1 0	0 1 1 0 1 0 0 1 0 0 1 1
0 0 1 0 0 0 0 0 0 0 0 0 1 0 1	1 0 0 1 0 1 1 0 1 1 0 1
0 0 1 0 0 1 1 1 0 0 0 0 0 0 0	0 0 1 0 0 1 1 1 0 0 0 0
0 0 1 0 0 0 1 1 1 0 0 0 0 0 0	0 0 1 0 0 0 1 1 1 0 0 0
0 0 1 0 0 0 0 1 1 1 0 0 0 0 0	0 0 1 0 0 0 0 1 1 1 0 0
0 0 1 0 0 0 0 0 1 1 1 0 0 0 0	0 0 1 0 0 0 0 0 1 1 1 0
0 0 1 0 0 0 0 0 0 1 1 1 0 0 0	0 0 1 0 0 0 0 0 0 1 1 1
0 0 1 0 0 0 0 0 0 0 1 1 1 0 0	1 0 1 1 0 0 1 0 0 1 1 1
0 0 1 0 0 0 0 0 0 0 0 1 1 1 0	1 1 1 1 1 0 1 1 0 1 1 1
0 0 1 0 0 0 0 0 0 0 0 0 1 1 1	1 1 0 1 1 1 1 1 1 1 1 1
0 0 1 1 0 0 0 0 0 0 0 0 0 0 0	0 0 1 1 0 0 0 0 0 0 0 0
0 0 1 1 0 1 0 0 0 0 0 0 0 0 0	0 0 1 1 0 1 0 0 0 0 0 0
0 0 1 1 0 0 1 0 0 0 0 0 0 0 0	0 0 1 1 0 0 1 0 0 0 0 0
0 0 1 1 0 0 0 1 0 0 0 0 0 0 0	0 0 1 1 0 0 0 1 0 0 0 0
0 0 1 1 0 0 0 0 1 0 0 0 0 0 0	0 0 1 1 0 0 0 0 1 0 0 0
0 0 1 1 0 0 0 0 0 1 0 0 0 0 0	0 0 1 1 0 0 0 0 0 1 0 0
0 0 1 1 0 0 0 0 0 0 1 0 0 0 0	0 0 1 1 0 0 0 0 0 0 1 0
0 0 1 1 0 0 0 0 0 0 0 1 0 0 0	0 0 1 1 0 0 0 0 0 0 0 1
0 0 1 1 0 0 0 0 0 0 0 0 1 0 0	1 0 1 0 0 0 1 0 0 1 0 0
0 0 1 1 0 0 0 0 0 0 0 0 0 1 0	0 1 1 1 1 0 0 1 0 0 1 0
0 0 1 1 0 0 0 0 0 0 0 0 0 0 1	0 0 0 1 0 1 0 0 1 0 0 1
0 0 1 1 0 1 1 0 0 0 0 0 0 0 0	0 0 1 1 0 1 1 0 0 0 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 1 1 0 0 1 1 0 0 0 0 0 0 0	0 0 1 1 0 0 1 1 0 0 0 0
0 0 1 1 0 0 0 1 1 0 0 0 0 0 0	0 0 1 1 0 0 0 1 1 0 0 0
0 0 1 1 0 0 0 0 1 1 0 0 0 0 0	0 0 1 1 0 0 0 0 1 1 0 0
0 0 1 1 0 0 0 0 0 1 1 0 0 0 0	0 0 1 1 0 0 0 0 0 1 1 0
0 0 1 1 0 0 0 0 0 0 1 1 0 0 0	0 0 1 1 0 0 0 0 0 0 1 1
0 0 1 1 0 0 0 0 0 0 0 1 1 0 0	1 0 1 0 0 0 1 0 0 1 0 1
0 0 1 1 0 0 0 0 0 0 0 0 1 1 0	1 1 1 0 1 0 1 1 0 1 1 0
0 0 1 1 0 0 0 0 0 0 0 0 0 1 1	0 1 0 1 1 1 0 1 1 0 1 1
0 0 1 1 0 1 0 1 0 0 0 0 0 0 0	0 0 1 1 0 1 0 1 0 0 0 0
0 0 1 1 0 0 1 0 1 0 0 0 0 0 0	0 0 1 1 0 0 1 0 1 0 0 0
0 0 1 1 0 0 0 1 0 1 0 0 0 0 0	0 0 1 1 0 0 0 1 0 1 0 0
0 0 1 1 0 0 0 0 1 0 1 0 0 0 0	0 0 1 1 0 0 0 0 1 0 1 0
0 0 1 1 0 0 0 0 0 1 0 1 0 0 0	0 0 1 1 0 0 0 0 0 1 0 1
0 0 1 1 0 0 0 0 0 0 1 0 1 0 0	1 0 1 0 0 0 1 0 0 1 1 0
0 0 1 1 0 0 0 0 0 0 0 1 0 1 0	0 1 1 1 1 0 0 1 0 0 1 1
0 0 1 1 0 0 0 0 0 0 0 0 1 0 1	1 0 0 0 0 1 1 0 1 1 0 1
0 0 1 1 0 1 1 1 0 0 0 0 0 0 0	0 0 1 1 0 1 1 1 0 0 0 0
0 0 1 1 0 0 1 1 1 0 0 0 0 0 0	0 0 1 1 0 0 1 1 1 0 0 0
0 0 1 1 0 0 0 1 1 1 0 0 0 0 0	0 0 1 1 0 0 0 1 1 1 0 0
0 0 1 1 0 0 0 0 1 1 1 0 0 0 0	0 0 1 1 0 0 0 0 1 1 1 0
0 0 1 1 0 0 0 0 0 1 1 1 0 0 0	0 0 1 1 0 0 0 0 0 1 1 1
0 0 1 1 0 0 0 0 0 0 1 1 1 0 0	1 0 1 0 0 0 1 0 0 1 1 1
0 0 1 1 0 0 0 0 0 0 0 1 1 1 0	1 1 1 0 1 0 1 1 0 1 1 1
0 0 1 1 0 0 0 0 0 0 0 0 1 1 1	1 1 0 0 1 1 1 1 1 1 1 1
0 0 1 0 1 0 0 0 0 0 0 0 0 0 0	0 0 1 0 1 0 0 0 0 0 0 0
0 0 1 0 1 1 0 0 0 0 0 0 0 0 0	0 0 1 0 1 1 0 0 0 0 0 0
0 0 1 0 1 0 1 0 0 0 0 0 0 0 0	0 0 1 0 1 0 1 0 0 0 0 0
0 0 1 0 1 0 0 1 0 0 0 0 0 0 0	0 0 1 0 1 0 0 1 0 0 0 0
0 0 1 0 1 0 0 0 1 0 0 0 0 0 0	0 0 1 0 1 0 0 0 1 0 0 0
0 0 1 0 1 0 0 0 0 1 0 0 0 0 0	0 0 1 0 1 0 0 0 0 1 0 0
0 0 1 0 1 0 0 0 0 0 1 0 0 0 0	0 0 1 0 1 0 0 0 0 0 1 0
0 0 1 0 1 0 0 0 0 0 0 1 0 0 0	0 0 1 0 1 0 0 0 0 0 0 1
0 0 1 0 1 0 0 0 0 0 0 0 1 0 0	1 0 1 1 1 0 1 0 0 1 0 0
0 0 1 0 1 0 0 0 0 0 0 0 0 1 0	0 1 1 0 0 0 0 1 0 0 1 0
0 0 1 0 1 0 0 0 0 0 0 0 0 0 1	0 0 0 0 1 1 0 0 1 0 0 1
0 0 1 0 1 1 1 0 0 0 0 0 0 0 0	0 0 1 0 1 1 1 0 0 0 0 0
0 0 1 0 1 0 1 1 0 0 0 0 0 0 0	0 0 1 0 1 0 1 1 0 0 0 0
0 0 1 0 1 0 0 1 1 0 0 0 0 0 0	0 0 1 0 1 0 0 1 1 0 0 0
0 0 1 0 1 0 0 0 1 1 0 0 0 0 0	0 0 1 0 1 0 0 0 1 1 0 0

Error vectors	Syndromes
0 0 1 0 1 0 0 0 0 1 1 0 0 0 0	0 0 1 0 1 0 0 0 0 1 1 0
0 0 1 0 1 0 0 0 0 0 1 1 0 0 0	0 0 1 0 1 0 0 0 0 0 1 1
0 0 1 0 1 0 0 0 0 0 0 1 1 0 0	1 0 1 1 1 0 1 0 0 1 0 1
0 0 1 0 1 0 0 0 0 0 0 0 1 1 0	1 1 1 1 0 0 1 1 0 1 1 0
0 0 1 0 1 0 0 0 0 0 0 0 0 1 1	0 1 0 0 0 1 0 1 1 0 1 1
0 0 1 0 1 1 0 1 0 0 0 0 0 0 0	0 0 1 0 1 1 0 1 0 0 0 0
0 0 1 0 1 0 1 0 1 0 0 0 0 0 0	0 0 1 0 1 0 1 0 1 0 0 0
0 0 1 0 1 0 0 1 0 1 0 0 0 0 0	0 0 1 0 1 0 0 1 0 1 0 0
0 0 1 0 1 0 0 0 1 0 1 0 0 0 0	0 0 1 0 1 0 0 0 1 0 1 0
0 0 1 0 1 0 0 0 0 1 0 1 0 0 0	0 0 1 0 1 0 0 0 0 1 0 1
0 0 1 0 1 0 0 0 0 0 1 0 1 0 0	1 0 1 1 1 0 1 0 0 1 1 0
0 0 1 0 1 0 0 0 0 0 0 1 0 1 0	0 1 1 0 0 0 0 1 0 0 1 1
0 0 1 0 1 0 0 0 0 0 0 0 1 0 1	1 0 0 1 1 1 1 0 1 1 0 1
0 0 1 0 1 1 1 1 0 0 0 0 0 0 0	0 0 1 0 1 1 1 1 0 0 0 0
0 0 1 0 1 0 1 1 1 0 0 0 0 0 0	0 0 1 0 1 0 1 1 1 0 0 0
0 0 1 0 1 0 0 1 1 1 0 0 0 0 0	0 0 1 0 1 0 0 1 1 1 0 0
0 0 1 0 1 0 0 0 1 1 1 0 0 0 0	0 0 1 0 1 0 0 0 1 1 1 0
0 0 1 0 1 0 0 0 0 1 1 1 0 0 0	0 0 1 0 1 0 0 0 0 1 1 1
0 0 1 0 1 0 0 0 0 0 1 1 1 0 0	1 0 1 1 1 0 1 0 0 1 1 1
0 0 1 0 1 0 0 0 0 0 0 1 1 1 0	1 1 1 1 0 0 1 1 0 1 1 1
0 0 1 0 1 0 0 0 0 0 0 0 1 1 1	1 1 0 1 0 1 1 1 1 1 1 1
0 0 1 1 1 0 0 0 0 0 0 0 0 0 0	0 0 1 1 1 0 0 0 0 0 0 0
0 0 1 1 1 1 0 0 0 0 0 0 0 0 0	0 0 1 1 1 1 0 0 0 0 0 0
0 0 1 1 1 0 1 0 0 0 0 0 0 0 0	0 0 1 1 1 0 1 0 0 0 0 0
0 0 1 1 1 0 0 1 0 0 0 0 0 0 0	0 0 1 1 1 0 0 1 0 0 0 0
0 0 1 1 1 0 0 0 1 0 0 0 0 0 0	0 0 1 1 1 0 0 0 1 0 0 0
0 0 1 1 1 0 0 0 0 1 0 0 0 0 0	0 0 1 1 1 0 0 0 0 1 0 0
0 0 1 1 1 0 0 0 0 0 1 0 0 0 0	0 0 1 1 1 0 0 0 0 0 1 0
0 0 1 1 1 0 0 0 0 0 0 1 0 0 0	0 0 1 1 1 0 0 0 0 0 0 1
0 0 1 1 1 0 0 0 0 0 0 0 1 0 0	1 0 1 0 1 0 1 0 0 1 0 0
0 0 1 1 1 0 0 0 0 0 0 0 0 1 0	0 1 1 1 0 0 0 1 0 0 1 0
0 0 1 1 1 0 0 0 0 0 0 0 0 0 1	0 0 0 1 1 1 0 0 1 0 0 1
0 0 1 1 1 1 1 0 0 0 0 0 0 0 0	0 0 1 1 1 1 1 0 0 0 0 0
0 0 1 1 1 0 1 1 0 0 0 0 0 0 0	0 0 1 1 1 0 1 1 0 0 0 0
0 0 1 1 1 0 0 1 1 0 0 0 0 0 0	0 0 1 1 1 0 0 1 1 0 0 0
0 0 1 1 1 0 0 0 1 1 0 0 0 0 0	0 0 1 1 1 0 0 0 1 1 0 0
0 0 1 1 1 0 0 0 0 1 1 0 0 0 0	0 0 1 1 1 0 0 0 0 1 1 0
0 0 1 1 1 0 0 0 0 0 1 1 0 0 0	0 0 1 1 1 0 0 0 0 0 1 1
0 0 1 1 1 0 0 0 0 0 0 1 1 0 0	1 0 1 0 1 0 1 0 0 1 0 1

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 1 1 1 0 0 0 0 0 0 0 1 1 0	1 1 1 0 0 0 1 1 0 1 1 0
0 0 1 1 1 0 0 0 0 0 0 0 0 1 1	0 1 0 1 0 1 0 1 1 0 1 1
0 0 1 1 1 1 0 1 0 0 0 0 0 0 0	0 0 1 1 1 1 0 1 0 0 0 0
0 0 1 1 1 0 1 0 1 0 0 0 0 0 0	0 0 1 1 1 0 1 0 1 0 0 0
0 0 1 1 1 0 0 1 0 1 0 0 0 0 0	0 0 1 1 1 0 0 1 0 1 0 0
0 0 1 1 1 0 0 0 1 0 1 0 0 0 0	0 0 1 1 1 0 0 0 1 0 1 0
0 0 1 1 1 0 0 0 0 1 0 1 0 0 0	0 0 1 1 1 0 0 0 0 1 0 1
0 0 1 1 1 0 0 0 0 0 1 0 1 0 0	1 0 1 0 1 0 1 0 0 1 1 0
0 0 1 1 1 0 0 0 0 0 0 1 0 1 0	0 1 1 1 0 0 0 1 0 0 1 1
0 0 1 1 1 0 0 0 0 0 0 0 1 0 1	1 0 0 0 1 1 1 0 1 1 0 1
0 0 1 1 1 1 1 1 0 0 0 0 0 0 0	0 0 1 1 1 1 1 1 0 0 0 0
0 0 1 1 1 0 1 1 1 0 0 0 0 0 0	0 0 1 1 1 0 1 1 1 0 0 0
0 0 1 1 1 0 0 1 1 1 0 0 0 0 0	0 0 1 1 1 0 0 1 1 1 0 0
0 0 1 1 1 0 0 0 1 1 1 0 0 0 0	0 0 1 1 1 0 0 0 1 1 1 0
0 0 1 1 1 0 0 0 0 1 1 1 0 0 0	0 0 1 1 1 0 0 0 0 1 1 1
0 0 1 1 1 0 0 0 0 0 1 1 1 0 0	1 0 1 0 1 0 1 0 0 1 1 1
0 0 1 1 1 0 0 0 0 0 0 1 1 1 0	1 1 1 0 0 0 1 1 0 1 1 1
0 0 1 1 1 0 0 0 0 0 0 0 1 1 1	1 1 0 0 0 1 1 1 1 1 1 1
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0	0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 1 0 0 1 0 0 0 0 0 0 0 0	0 0 0 1 0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 1 0 0 0 0 0 0 0	0 0 0 1 0 0 0 1 0 0 0 0
0 0 0 1 0 0 0 0 1 0 0 0 0 0 0	0 0 0 1 0 0 0 0 1 0 0 0
0 0 0 1 0 0 0 0 0 1 0 0 0 0 0	0 0 0 1 0 0 0 0 0 1 0 0
0 0 0 1 0 0 0 0 0 0 1 0 0 0 0	0 0 0 1 0 0 0 0 0 0 1 0
0 0 0 1 0 0 0 0 0 0 0 1 0 0 0	0 0 0 1 0 0 0 0 0 0 0 1
0 0 0 1 0 0 0 0 0 0 0 0 1 0 0	1 0 0 0 0 0 1 0 0 1 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 1 0	0 1 0 1 1 0 0 1 0 0 1 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 1	0 0 1 1 0 1 0 0 1 0 0 1
0 0 0 1 0 0 1 1 0 0 0 0 0 0 0	0 0 0 1 0 0 1 1 0 0 0 0
0 0 0 1 0 0 0 1 1 0 0 0 0 0 0	0 0 0 1 0 0 0 1 1 0 0 0
0 0 0 1 0 0 0 0 1 1 0 0 0 0 0	0 0 0 1 0 0 0 0 1 1 0 0
0 0 0 1 0 0 0 0 0 1 1 0 0 0 0	0 0 0 1 0 0 0 0 0 1 1 0
0 0 0 1 0 0 0 0 0 0 1 1 0 0 0	0 0 0 1 0 0 0 0 0 0 1 1
0 0 0 1 0 0 0 0 0 0 0 1 1 0 0	1 0 0 0 0 0 1 0 0 1 0 1
0 0 0 1 0 0 0 0 0 0 0 0 1 1 0	1 1 0 0 1 0 1 1 0 1 1 0
0 0 0 1 0 0 0 0 0 0 0 0 0 1 1	0 1 1 1 1 1 0 1 1 0 1 1
0 0 0 1 0 0 1 0 1 0 0 0 0 0 0	0 0 0 1 0 0 1 0 1 0 0 0
0 0 0 1 0 0 0 1 0 1 0 0 0 0 0	0 0 0 1 0 0 0 1 0 1 0 0
0 0 0 1 0 0 0 0 1 0 1 0 0 0 0	0 0 0 1 0 0 0 0 1 0 1 0

Error vectors	Syndromes
0 0 0 1 0 0 0 0 0 1 0 1 0 0 0	0 0 0 1 0 0 0 0 0 1 0 1
0 0 0 1 0 0 0 0 0 0 1 0 1 0 0	1 0 0 0 0 0 1 0 0 1 1 0
0 0 0 1 0 0 0 0 0 0 0 1 0 1 0	0 1 0 1 1 0 0 1 0 0 1 1
0 0 0 1 0 0 0 0 0 0 0 0 1 0 1	1 0 1 0 0 1 1 0 1 1 0 1
0 0 0 1 0 0 1 1 1 0 0 0 0 0 0	0 0 0 1 0 0 1 1 1 0 0 0
0 0 0 1 0 0 0 1 1 1 0 0 0 0 0	0 0 0 1 0 0 0 1 1 1 0 0
0 0 0 1 0 0 0 0 1 1 1 0 0 0 0	0 0 0 1 0 0 0 0 1 1 1 0
0 0 0 1 0 0 0 0 0 1 1 1 0 0 0	0 0 0 1 0 0 0 0 0 1 1 1
0 0 0 1 0 0 0 0 0 0 1 1 1 0 0	1 0 0 0 0 0 1 0 0 1 1 1
0 0 0 1 0 0 0 0 0 0 0 1 1 1 0	1 1 0 0 1 0 1 1 0 1 1 1
0 0 0 1 0 0 0 0 0 0 0 0 1 1 1	1 1 1 0 1 1 1 1 1 1 1 1
0 0 0 1 1 0 0 0 0 0 0 0 0 0 0	0 0 0 1 1 0 0 0 0 0 0 0
0 0 0 1 1 0 1 0 0 0 0 0 0 0 0	0 0 0 1 1 0 1 0 0 0 0 0
0 0 0 1 1 0 0 1 0 0 0 0 0 0 0	0 0 0 1 1 0 0 1 0 0 0 0
0 0 0 1 1 0 0 0 1 0 0 0 0 0 0	0 0 0 1 1 0 0 0 1 0 0 0
0 0 0 1 1 0 0 0 0 1 0 0 0 0 0	0 0 0 1 1 0 0 0 0 1 0 0
0 0 0 1 1 0 0 0 0 0 1 0 0 0 0	0 0 0 1 1 0 0 0 0 0 1 0
0 0 0 1 1 0 0 0 0 0 0 1 0 0 0	0 0 0 1 1 0 0 0 0 0 0 1
0 0 0 1 1 0 0 0 0 0 0 0 1 0 0	1 0 0 0 1 0 1 0 0 1 0 0
0 0 0 1 1 0 0 0 0 0 0 0 0 1 0	0 1 0 1 0 0 0 1 0 0 1 0
0 0 0 1 1 0 0 0 0 0 0 0 0 0 1	0 0 1 1 1 1 0 0 1 0 0 1
0 0 0 1 1 0 1 1 0 0 0 0 0 0 0	0 0 0 1 1 0 1 1 0 0 0 0
0 0 0 1 1 0 0 1 1 0 0 0 0 0 0	0 0 0 1 1 0 0 1 1 0 0 0
0 0 0 1 1 0 0 0 1 1 0 0 0 0 0	0 0 0 1 1 0 0 0 1 1 0 0
0 0 0 1 1 0 0 0 0 1 1 0 0 0 0	0 0 0 1 1 0 0 0 0 1 1 0
0 0 0 1 1 0 0 0 0 0 1 1 0 0 0	0 0 0 1 1 0 0 0 0 0 1 1
0 0 0 1 1 0 0 0 0 0 0 1 1 0 0	1 0 0 0 1 0 1 0 0 1 0 1
0 0 0 1 1 0 0 0 0 0 0 0 1 1 0	1 1 0 0 0 0 1 1 0 1 1 0
0 0 0 1 1 0 0 0 0 0 0 0 0 1 1	0 1 1 1 0 1 0 1 1 0 1 1
0 0 0 1 1 0 1 0 1 0 0 0 0 0 0	0 0 0 1 1 0 1 0 1 0 0 0
0 0 0 1 1 0 0 1 0 1 0 0 0 0 0	0 0 0 1 1 0 0 1 0 1 0 0
0 0 0 1 1 0 0 0 1 0 1 0 0 0 0	0 0 0 1 1 0 0 0 1 0 1 0
0 0 0 1 1 0 0 0 0 1 0 1 0 0 0	0 0 0 1 1 0 0 0 0 1 0 1
0 0 0 1 1 0 0 0 0 0 1 0 1 0 0	1 0 0 0 1 0 1 0 0 1 1 0
0 0 0 1 1 0 0 0 0 0 0 1 0 1 0	0 1 0 1 0 0 0 1 0 0 1 1
0 0 0 1 1 0 0 0 0 0 0 0 1 0 1	1 0 1 0 1 1 1 0 1 1 0 1
0 0 0 1 1 0 1 1 1 0 0 0 0 0 0	0 0 0 1 1 0 1 1 1 0 0 0
0 0 0 1 1 0 0 1 1 1 0 0 0 0 0	0 0 0 1 1 0 0 1 1 1 0 0
0 0 0 1 1 0 0 0 1 1 1 0 0 0 0	0 0 0 1 1 0 0 0 1 1 1 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 1 1 0 0 0 0 1 1 1 0 0 0	0 0 0 1 1 0 0 0 0 1 1 1
0 0 0 1 1 0 0 0 0 0 1 1 1 0 0	1 0 0 0 1 0 1 0 0 1 1 1
0 0 0 1 1 0 0 0 0 0 0 1 1 1 0	1 1 0 0 0 0 1 1 0 1 1 1
0 0 0 1 1 0 0 0 0 0 0 0 1 1 1	1 1 1 0 0 1 1 1 1 1 1 1
0 0 0 1 0 1 0 0 0 0 0 0 0 0 0	0 0 0 1 0 1 0 0 0 0 0 0
0 0 0 1 0 1 1 0 0 0 0 0 0 0 0	0 0 0 1 0 1 1 0 0 0 0 0
0 0 0 1 0 1 0 1 0 0 0 0 0 0 0	0 0 0 1 0 1 0 1 0 0 0 0
0 0 0 1 0 1 0 0 1 0 0 0 0 0 0	0 0 0 1 0 1 0 0 1 0 0 0
0 0 0 1 0 1 0 0 0 1 0 0 0 0 0	0 0 0 1 0 1 0 0 0 1 0 0
0 0 0 1 0 1 0 0 0 0 1 0 0 0 0	0 0 0 1 0 1 0 0 0 0 1 0
0 0 0 1 0 1 0 0 0 0 0 1 0 0 0	0 0 0 1 0 1 0 0 0 0 0 1
0 0 0 1 0 1 0 0 0 0 0 0 1 0 0	1 0 0 0 0 1 1 0 0 1 0 0
0 0 0 1 0 1 0 0 0 0 0 0 0 1 0	0 1 0 1 1 1 0 1 0 0 1 0
0 0 0 1 0 1 0 0 0 0 0 0 0 0 1	0 0 1 1 0 0 0 0 1 0 0 1
0 0 0 1 0 1 1 1 0 0 0 0 0 0 0	0 0 0 1 0 1 1 1 0 0 0 0
0 0 0 1 0 1 0 1 1 0 0 0 0 0 0	0 0 0 1 0 1 0 1 1 0 0 0
0 0 0 1 0 1 0 0 1 1 0 0 0 0 0	0 0 0 1 0 1 0 0 1 1 0 0
0 0 0 1 0 1 0 0 0 1 1 0 0 0 0	0 0 0 1 0 1 0 0 0 1 1 0
0 0 0 1 0 1 0 0 0 0 1 1 0 0 0	0 0 0 1 0 1 0 0 0 0 1 1
0 0 0 1 0 1 0 0 0 0 0 1 1 0 0	1 0 0 0 0 1 1 0 0 1 0 1
0 0 0 1 0 1 0 0 0 0 0 0 1 1 0	1 1 0 0 1 1 1 1 0 1 1 0
0 0 0 1 0 1 0 0 0 0 0 0 0 1 1	0 1 1 1 1 0 0 1 1 0 1 1
0 0 0 1 0 1 1 0 1 0 0 0 0 0 0	0 0 0 1 0 1 1 0 1 0 0 0
0 0 0 1 0 1 0 1 0 1 0 0 0 0 0	0 0 0 1 0 1 0 1 0 1 0 0
0 0 0 1 0 1 0 0 1 0 1 0 0 0 0	0 0 0 1 0 1 0 0 1 0 1 0
0 0 0 1 0 1 0 0 0 1 0 1 0 0 0	0 0 0 1 0 1 0 0 0 1 0 1
0 0 0 1 0 1 0 0 0 0 1 0 1 0 0	1 0 0 0 0 1 1 0 0 1 1 0
0 0 0 1 0 1 0 0 0 0 0 1 0 1 0	0 1 0 1 1 1 0 1 0 0 1 1
0 0 0 1 0 1 0 0 0 0 0 0 1 0 1	1 0 1 0 0 0 1 0 1 1 0 1
0 0 0 1 0 1 1 1 1 0 0 0 0 0 0	0 0 0 1 0 1 1 1 1 0 0 0
0 0 0 1 0 1 0 1 1 1 0 0 0 0 0	0 0 0 1 0 1 0 1 1 1 0 0
0 0 0 1 0 1 0 0 1 1 1 0 0 0 0	0 0 0 1 0 1 0 0 1 1 1 0
0 0 0 1 0 1 0 0 0 1 1 1 0 0 0	0 0 0 1 0 1 0 0 0 1 1 1
0 0 0 1 0 1 0 0 0 0 1 1 1 0 0	1 0 0 0 0 1 1 0 0 1 1 1
0 0 0 1 0 1 0 0 0 0 0 1 1 1 0	1 1 0 0 1 1 1 1 0 1 1 1
0 0 0 1 0 1 0 0 0 0 0 0 1 1 1	1 1 1 0 1 0 1 1 1 1 1 1
0 0 0 1 1 1 0 0 0 0 0 0 0 0 0	0 0 0 1 1 1 0 0 0 0 0 0
0 0 0 1 1 1 1 0 0 0 0 0 0 0 0	0 0 0 1 1 1 1 0 0 0 0 0
0 0 0 1 1 1 0 1 0 0 0 0 0 0 0	0 0 0 1 1 1 0 1 0 0 0 0

Error vectors	Syndromes
0 0 0 1 1 1 0 0 1 0 0 0 0 0 0	0 0 0 1 1 1 0 0 1 0 0 0
0 0 0 1 1 1 0 0 0 1 0 0 0 0 0	0 0 0 1 1 1 0 0 0 1 0 0
0 0 0 1 1 1 0 0 0 0 1 0 0 0 0	0 0 0 1 1 1 0 0 0 0 1 0
0 0 0 1 1 1 0 0 0 0 0 1 0 0 0	0 0 0 1 1 1 0 0 0 0 0 1
0 0 0 1 1 1 0 0 0 0 0 0 1 0 0	1 0 0 0 1 1 1 0 0 1 0 0
0 0 0 1 1 1 0 0 0 0 0 0 0 1 0	0 1 0 1 0 1 0 1 0 0 1 0
0 0 0 1 1 1 0 0 0 0 0 0 0 0 1	0 0 1 1 1 0 0 0 1 0 0 1
0 0 0 1 1 1 1 1 0 0 0 0 0 0 0	0 0 0 1 1 1 1 1 0 0 0 0
0 0 0 1 1 1 0 1 1 0 0 0 0 0 0	0 0 0 1 1 1 0 1 1 0 0 0
0 0 0 1 1 1 0 0 1 1 0 0 0 0 0	0 0 0 1 1 1 0 0 1 1 0 0
0 0 0 1 1 1 0 0 0 1 1 0 0 0 0	0 0 0 1 1 1 0 0 0 1 1 0
0 0 0 1 1 1 0 0 0 0 1 1 0 0 0	0 0 0 1 1 1 0 0 0 0 1 1
0 0 0 1 1 1 0 0 0 0 0 1 1 0 0	1 0 0 0 1 1 1 0 0 1 0 1
0 0 0 1 1 1 0 0 0 0 0 0 1 1 0	1 1 0 0 0 1 1 1 0 1 1 0
0 0 0 1 1 1 0 0 0 0 0 0 0 1 1	0 1 1 1 0 0 0 1 1 0 1 1
0 0 0 1 1 1 1 0 1 0 0 0 0 0 0	0 0 0 1 1 1 1 0 1 0 0 0
0 0 0 1 1 1 0 1 0 1 0 0 0 0 0	0 0 0 1 1 1 0 1 0 1 0 0
0 0 0 1 1 1 0 0 1 0 1 0 0 0 0	0 0 0 1 1 1 0 0 1 0 1 0
0 0 0 1 1 1 0 0 0 1 0 1 0 0 0	0 0 0 1 1 1 0 0 0 1 0 1
0 0 0 1 1 1 0 0 0 0 1 0 1 0 0	1 0 0 0 1 1 1 0 0 1 1 0
0 0 0 1 1 1 0 0 0 0 0 1 0 1 0	0 1 0 1 0 1 0 1 0 0 1 1
0 0 0 1 1 1 0 0 0 0 0 0 1 0 1	1 0 1 0 1 0 1 0 1 1 0 1
0 0 0 1 1 1 1 1 1 0 0 0 0 0 0	0 0 0 1 1 1 1 1 1 0 0 0
0 0 0 1 1 1 0 1 1 1 0 0 0 0 0	0 0 0 1 1 1 0 1 1 1 0 0
0 0 0 1 1 1 0 0 1 1 1 0 0 0 0	0 0 0 1 1 1 0 0 1 1 1 0
0 0 0 1 1 1 0 0 0 1 1 1 0 0 0	0 0 0 1 1 1 0 0 0 1 1 1
0 0 0 1 1 1 0 0 0 0 1 1 1 0 0	1 0 0 0 1 1 1 0 0 1 1 1
0 0 0 1 1 1 0 0 0 0 0 1 1 1 0	1 1 0 0 0 1 1 1 0 1 1 1
0 0 0 1 1 1 0 0 0 0 0 0 1 1 1	1 1 1 0 0 0 1 1 1 1 1 1
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0	0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 1 0 0 1 0 0 0 0 0 0 0	0 0 0 0 1 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0 1 0 0 0 0 0 0	0 0 0 0 1 0 0 0 1 0 0 0
0 0 0 0 1 0 0 0 0 1 0 0 0 0 0	0 0 0 0 1 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0 0 0 1 0 0 0 0	0 0 0 0 1 0 0 0 0 0 1 0
0 0 0 0 1 0 0 0 0 0 0 1 0 0 0	0 0 0 0 1 0 0 0 0 0 0 1
0 0 0 0 1 0 0 0 0 0 0 0 1 0 0	1 0 0 1 1 0 1 0 0 1 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 1 0	0 1 0 0 0 0 0 1 0 0 1 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 1	0 0 1 0 1 1 0 0 1 0 0 1
0 0 0 0 1 0 0 1 1 0 0 0 0 0 0	0 0 0 0 1 0 0 1 1 0 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 0 1 0 0 0 1 1 0 0 0 0 0	0 0 0 0 1 0 0 0 1 1 0 0
0 0 0 0 1 0 0 0 0 1 1 0 0 0 0	0 0 0 0 1 0 0 0 0 1 1 0
0 0 0 0 1 0 0 0 0 0 1 1 0 0 0	0 0 0 0 1 0 0 0 0 0 1 1
0 0 0 0 1 0 0 0 0 0 0 1 1 0 0	1 0 0 1 1 0 1 0 0 1 0 1
0 0 0 0 1 0 0 0 0 0 0 0 1 1 0	1 1 0 1 0 0 1 1 0 1 1 0
0 0 0 0 1 0 0 0 0 0 0 0 0 1 1	0 1 1 0 0 1 0 1 1 0 1 1
0 0 0 0 1 0 0 1 0 1 0 0 0 0 0	0 0 0 0 1 0 0 1 0 1 0 0
0 0 0 0 1 0 0 0 1 0 1 0 0 0 0	0 0 0 0 1 0 0 0 1 0 1 0
0 0 0 0 1 0 0 0 0 1 0 1 0 0 0	0 0 0 0 1 0 0 0 0 1 0 1
0 0 0 0 1 0 0 0 0 0 1 0 1 0 0	1 0 0 1 1 0 1 0 0 1 1 0
0 0 0 0 1 0 0 0 0 0 0 1 0 1 0	0 1 0 0 0 0 0 1 0 0 1 1
0 0 0 0 1 0 0 0 0 0 0 0 1 0 1	1 0 1 1 1 1 1 0 1 1 0 1
0 0 0 0 1 0 0 1 1 1 0 0 0 0 0	0 0 0 0 1 0 0 1 1 1 0 0
0 0 0 0 1 0 0 0 1 1 1 0 0 0 0	0 0 0 0 1 0 0 0 1 1 1 0
0 0 0 0 1 0 0 0 0 1 1 1 0 0 0	0 0 0 0 1 0 0 0 0 1 1 1
0 0 0 0 1 0 0 0 0 0 1 1 1 0 0	1 0 0 1 1 0 1 0 0 1 1 1
0 0 0 0 1 0 0 0 0 0 0 1 1 1 0	1 1 0 1 0 0 1 1 0 1 1 1
0 0 0 0 1 0 0 0 0 0 0 0 1 1 1	1 1 1 1 0 1 1 1 1 1 1 1
0 0 0 0 1 1 0 0 0 0 0 0 0 0 0	0 0 0 0 1 1 0 0 0 0 0 0
0 0 0 0 1 1 0 1 0 0 0 0 0 0 0	0 0 0 0 1 1 0 1 0 0 0 0
0 0 0 0 1 1 0 0 1 0 0 0 0 0 0	0 0 0 0 1 1 0 0 1 0 0 0
0 0 0 0 1 1 0 0 0 1 0 0 0 0 0	0 0 0 0 1 1 0 0 0 1 0 0
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0 0 0 0 1 1 0 0 0 0 0 1 0 0 0	0 0 0 0 1 1 0 0 0 0 0 1
0 0 0 0 1 1 0 0 0 0 0 0 1 0 0	1 0 0 1 1 1 1 0 0 1 0 0
0 0 0 0 1 1 0 0 0 0 0 0 0 1 0	0 1 0 0 0 1 0 1 0 0 1 0
0 0 0 0 1 1 0 0 0 0 0 0 0 0 1	0 0 1 0 1 0 0 0 1 0 0 1
0 0 0 0 1 1 0 1 1 0 0 0 0 0 0	0 0 0 0 1 1 0 1 1 0 0 0
0 0 0 0 1 1 0 0 1 1 0 0 0 0 0	0 0 0 0 1 1 0 0 1 1 0 0
0 0 0 0 1 1 0 0 0 1 1 0 0 0 0	0 0 0 0 1 1 0 0 0 1 1 0
0 0 0 0 1 1 0 0 0 0 1 1 0 0 0	0 0 0 0 1 1 0 0 0 0 1 1
0 0 0 0 1 1 0 0 0 0 0 1 1 0 0	1 0 0 1 1 1 1 0 0 1 0 1
0 0 0 0 1 1 0 0 0 0 0 0 1 1 0	1 1 0 1 0 1 1 1 0 1 1 0
0 0 0 0 1 1 0 0 0 0 0 0 0 1 1	0 1 1 0 0 0 0 1 1 0 1 1
0 0 0 0 1 1 0 1 0 1 0 0 0 0 0	0 0 0 0 1 1 0 1 0 1 0 0
0 0 0 0 1 1 0 0 1 0 1 0 0 0 0	0 0 0 0 1 1 0 0 1 0 1 0
0 0 0 0 1 1 0 0 0 1 0 1 0 0 0	0 0 0 0 1 1 0 0 0 1 0 1
0 0 0 0 1 1 0 0 0 0 1 0 1 0 0	1 0 0 1 1 1 1 0 0 1 1 0
0 0 0 0 1 1 0 0 0 0 0 1 0 1 0	0 1 0 0 0 1 0 1 0 0 1 1
0 0 0 0 1 1 0 0 0 0 0 0 1 0 1	1 0 1 1 1 0 1 0 1 1 0 1
0 0 0 0 1 1 0 1 1 1 0 0 0 0 0	0 0 0 0 1 1 0 1 1 1 0 0
0 0 0 0 1 1 0 0 1 1 1 0 0 0 0	0 0 0 0 1 1 0 0 1 1 1 0
0 0 0 0 1 1 0 0 0 1 1 1 0 0 0	0 0 0 0 1 1 0 0 0 1 1 1
0 0 0 0 1 1 0 0 0 0 1 1 1 0 0	1 0 0 1 1 1 1 0 0 1 1 1
0 0 0 0 1 1 0 0 0 0 0 1 1 1 0	1 1 0 1 0 1 1 1 0 1 1 1
0 0 0 0 1 1 0 0 0 0 0 0 1 1 1	1 1 1 1 0 0 1 1 1 1 1 1
0 0 0 0 1 0 1 0 0 0 0 0 0 0 0	0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 1 0 1 1 0 0 0 0 0 0 0	0 0 0 0 1 0 1 1 0 0 0 0
0 0 0 0 1 0 1 0 1 0 0 0 0 0 0	0 0 0 0 1 0 1 0 1 0 0 0
0 0 0 0 1 0 1 0 0 1 0 0 0 0 0	0 0 0 0 1 0 1 0 0 1 0 0
0 0 0 0 1 0 1 0 0 0 1 0 0 0 0	0 0 0 0 1 0 1 0 0 0 1 0
0 0 0 0 1 0 1 0 0 0 0 1 0 0 0	0 0 0 0 1 0 1 0 0 0 0 1
0 0 0 0 1 0 1 0 0 0 0 0 1 0 0	1 0 0 1 1 0 0 0 0 1 0 0
0 0 0 0 1 0 1 0 0 0 0 0 0 1 0	0 1 0 0 0 0 1 1 0 0 1 0
0 0 0 0 1 0 1 0 0 0 0 0 0 0 1	0 0 1 0 1 1 1 0 1 0 0 1
0 0 0 0 1 0 1 1 1 0 0 0 0 0 0	0 0 0 0 1 0 1 1 1 0 0 0
0 0 0 0 1 0 1 0 1 1 0 0 0 0 0	0 0 0 0 1 0 1 0 1 1 0 0
0 0 0 0 1 0 1 0 0 1 1 0 0 0 0	0 0 0 0 1 0 1 0 0 1 1 0
0 0 0 0 1 0 1 0 0 0 1 1 0 0 0	0 0 0 0 1 0 1 0 0 0 1 1
0 0 0 0 1 0 1 0 0 0 0 1 1 0 0	1 0 0 1 1 0 0 0 0 1 0 1
0 0 0 0 1 0 1 0 0 0 0 0 1 1 0	1 1 0 1 0 0 0 1 0 1 1 0
0 0 0 0 1 0 1 0 0 0 0 0 0 1 1	0 1 1 0 0 1 1 1 1 0 1 1
0 0 0 0 1 0 1 1 0 1 0 0 0 0 0	0 0 0 0 1 0 1 1 0 1 0 0
0 0 0 0 1 0 1 0 1 0 1 0 0 0 0	0 0 0 0 1 0 1 0 1 0 1 0
0 0 0 0 1 0 1 0 0 1 0 1 0 0 0	0 0 0 0 1 0 1 0 0 1 0 1
0 0 0 0 1 0 1 0 0 0 1 0 1 0 0	1 0 0 1 1 0 0 0 0 1 1 0
0 0 0 0 1 0 1 0 0 0 0 1 0 1 0	0 1 0 0 0 0 1 1 0 0 1 1
0 0 0 0 1 0 1 0 0 0 0 0 1 0 1	1 0 1 1 1 1 0 0 1 1 0 1
0 0 0 0 1 0 1 1 1 1 0 0 0 0 0	0 0 0 0 1 0 1 1 1 1 0 0
0 0 0 0 1 0 1 0 1 1 1 0 0 0 0	0 0 0 0 1 0 1 0 1 1 1 0
0 0 0 0 1 0 1 0 0 1 1 1 0 0 0	0 0 0 0 1 0 1 0 0 1 1 1
0 0 0 0 1 0 1 0 0 0 1 1 1 0 0	1 0 0 1 1 0 0 0 0 1 1 1
0 0 0 0 1 0 1 0 0 0 0 1 1 1 0	1 1 0 1 0 0 0 1 0 1 1 1
0 0 0 0 1 0 1 0 0 0 0 0 1 1 1	1 1 1 1 0 1 0 1 1 1 1 1
0 0 0 0 1 1 1 0 0 0 0 0 0 0 0	0 0 0 0 1 1 1 0 0 0 0 0
0 0 0 0 1 1 1 1 0 0 0 0 0 0 0	0 0 0 0 1 1 1 1 0 0 0 0
0 0 0 0 1 1 1 0 1 0 0 0 0 0 0	0 0 0 0 1 1 1 0 1 0 0 0
0 0 0 0 1 1 1 0 0 1 0 0 0 0 0	0 0 0 0 1 1 1 0 0 1 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 0 1 1 1 0 0 0 1 0 0 0 0	0 0 0 0 1 1 1 0 0 0 1 0
0 0 0 0 1 1 1 0 0 0 0 1 0 0 0	0 0 0 0 1 1 1 0 0 0 0 1
0 0 0 0 1 1 1 0 0 0 0 0 1 0 0	1 0 0 1 1 1 0 0 0 1 0 0
0 0 0 0 1 1 1 0 0 0 0 0 0 1 0	0 1 0 0 0 1 1 1 0 0 1 0
0 0 0 0 1 1 1 0 0 0 0 0 0 0 1	0 0 1 0 1 0 1 0 1 0 0 1
0 0 0 0 1 1 1 1 1 0 0 0 0 0 0	0 0 0 0 1 1 1 1 1 0 0 0
0 0 0 0 1 1 1 0 1 1 0 0 0 0 0	0 0 0 0 1 1 1 0 1 1 0 0
0 0 0 0 1 1 1 0 0 1 1 0 0 0 0	0 0 0 0 1 1 1 0 0 1 1 0
0 0 0 0 1 1 1 0 0 0 1 1 0 0 0	0 0 0 0 1 1 1 0 0 0 1 1
0 0 0 0 1 1 1 0 0 0 0 1 1 0 0	1 0 0 1 1 1 0 0 0 1 0 1
0 0 0 0 1 1 1 0 0 0 0 0 1 1 0	1 1 0 1 0 1 0 1 0 1 1 0
0 0 0 0 1 1 1 0 0 0 0 0 0 1 1	0 1 1 0 0 0 1 1 1 0 1 1
0 0 0 0 1 1 1 1 0 1 0 0 0 0 0	0 0 0 0 1 1 1 1 0 1 0 0
0 0 0 0 1 1 1 0 1 0 1 0 0 0 0	0 0 0 0 1 1 1 0 1 0 1 0
0 0 0 0 1 1 1 0 0 1 0 1 0 0 0	0 0 0 0 1 1 1 0 0 1 0 1
0 0 0 0 1 1 1 0 0 0 1 0 1 0 0	1 0 0 1 1 1 0 0 0 1 1 0
0 0 0 0 1 1 1 0 0 0 0 1 0 1 0	0 1 0 0 0 1 1 1 0 0 1 1
0 0 0 0 1 1 1 0 0 0 0 0 1 0 1	1 0 1 1 1 0 0 0 1 1 0 1
0 0 0 0 1 1 1 1 1 1 0 0 0 0 0	0 0 0 0 1 1 1 1 1 1 0 0
0 0 0 0 1 1 1 0 1 1 1 0 0 0 0	0 0 0 0 1 1 1 0 1 1 1 0
0 0 0 0 1 1 1 0 0 1 1 1 0 0 0	0 0 0 0 1 1 1 0 0 1 1 1
0 0 0 0 1 1 1 0 0 0 1 1 1 0 0	1 0 0 1 1 1 0 0 0 1 1 1
0 0 0 0 1 1 1 0 0 0 0 1 1 1 0	1 1 0 1 0 1 0 1 0 1 1 1
0 0 0 0 1 1 1 0 0 0 0 0 1 1 1	1 1 1 1 0 0 0 1 1 1 1 1
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0	0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 1 0 0 1 0 0 0 0 0 0	0 0 0 0 0 1 0 0 1 0 0 0
0 0 0 0 0 1 0 0 0 1 0 0 0 0 0	0 0 0 0 0 1 0 0 0 1 0 0
0 0 0 0 0 1 0 0 0 0 1 0 0 0 0	0 0 0 0 0 1 0 0 0 0 1 0
0 0 0 0 0 1 0 0 0 0 0 1 0 0 0	0 0 0 0 0 1 0 0 0 0 0 1
0 0 0 0 0 1 0 0 0 0 0 0 1 0 0	1 0 0 1 0 1 1 0 0 1 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 1 0	0 1 0 0 1 1 0 1 0 0 1 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 1	0 0 1 0 0 0 0 0 1 0 0 1
0 0 0 0 0 1 0 0 1 1 0 0 0 0 0	0 0 0 0 0 1 0 0 1 1 0 0
0 0 0 0 0 1 0 0 0 1 1 0 0 0 0	0 0 0 0 0 1 0 0 0 1 1 0
0 0 0 0 0 1 0 0 0 0 1 1 0 0 0	0 0 0 0 0 1 0 0 0 0 1 1
0 0 0 0 0 1 0 0 0 0 0 1 1 0 0	1 0 0 1 0 1 1 0 0 1 0 1
0 0 0 0 0 1 0 0 0 0 0 0 1 1 0	1 1 0 1 1 1 1 1 0 1 1 0
0 0 0 0 0 1 0 0 0 0 0 0 0 1 1	0 1 1 0 1 0 0 1 1 0 1 1
0 0 0 0 0 1 0 0 1 0 1 0 0 0 0	0 0 0 0 0 1 0 0 1 0 1 0

Error vectors	Syndromes
0 0 0 0 0 1 0 0 0 1 0 1 0 0 0	0 0 0 0 0 1 0 0 0 1 0 1
0 0 0 0 0 1 0 0 0 0 1 0 1 0 0	1 0 0 1 0 1 1 0 0 1 1 0
0 0 0 0 0 1 0 0 0 0 0 1 0 1 0	0 1 0 0 1 1 0 1 0 0 1 1
0 0 0 0 0 1 0 0 0 0 0 0 1 0 1	1 0 1 1 0 0 1 0 1 1 0 1
0 0 0 0 0 1 0 0 1 1 1 0 0 0 0	0 0 0 0 0 1 0 0 1 1 1 0
0 0 0 0 0 1 0 0 0 1 1 1 0 0 0	0 0 0 0 0 1 0 0 0 1 1 1
0 0 0 0 0 1 0 0 0 0 1 1 1 0 0	1 0 0 1 0 1 1 0 0 1 1 1
0 0 0 0 0 1 0 0 0 0 0 1 1 1 0	1 1 0 1 1 1 1 1 0 1 1 1
0 0 0 0 0 1 0 0 0 0 0 0 1 1 1	1 1 1 1 1 0 1 1 1 1 1 1
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0	0 0 0 0 0 1 1 0 0 0 0 0
0 0 0 0 0 1 1 0 1 0 0 0 0 0 0	0 0 0 0 0 1 1 0 1 0 0 0
0 0 0 0 0 1 1 0 0 1 0 0 0 0 0	0 0 0 0 0 1 1 0 0 1 0 0
0 0 0 0 0 1 1 0 0 0 1 0 0 0 0	0 0 0 0 0 1 1 0 0 0 1 0
0 0 0 0 0 1 1 0 0 0 0 1 0 0 0	0 0 0 0 0 1 1 0 0 0 0 1
0 0 0 0 0 1 1 0 0 0 0 0 1 0 0	1 0 0 1 0 1 0 0 0 1 0 0
0 0 0 0 0 1 1 0 0 0 0 0 0 1 0	0 1 0 0 1 1 1 1 0 0 1 0
0 0 0 0 0 1 1 0 0 0 0 0 0 0 1	0 0 1 0 0 0 1 0 1 0 0 1
0 0 0 0 0 1 1 0 1 1 0 0 0 0 0	0 0 0 0 0 1 1 0 1 1 0 0
0 0 0 0 0 1 1 0 0 1 1 0 0 0 0	0 0 0 0 0 1 1 0 0 1 1 0
0 0 0 0 0 1 1 0 0 0 1 1 0 0 0	0 0 0 0 0 1 1 0 0 0 1 1
0 0 0 0 0 1 1 0 0 0 0 1 1 0 0	1 0 0 1 0 1 0 0 0 1 0 1
0 0 0 0 0 1 1 0 0 0 0 0 1 1 0	1 1 0 1 1 1 0 1 0 1 1 0
0 0 0 0 0 1 1 0 0 0 0 0 0 1 1	0 1 1 0 1 0 1 1 1 0 1 1
0 0 0 0 0 1 1 0 1 0 1 0 0 0 0	0 0 0 0 0 1 1 0 1 0 1 0
0 0 0 0 0 1 1 0 0 1 0 1 0 0 0	0 0 0 0 0 1 1 0 0 1 0 1
0 0 0 0 0 1 1 0 0 0 1 0 1 0 0	1 0 0 1 0 1 0 0 0 1 1 0
0 0 0 0 0 1 1 0 0 0 0 1 0 1 0	0 1 0 0 1 1 1 1 0 0 1 1
0 0 0 0 0 1 1 0 0 0 0 0 1 0 1	1 0 1 1 0 0 0 0 1 1 0 1
0 0 0 0 0 1 1 0 1 1 1 0 0 0 0	0 0 0 0 0 1 1 0 1 1 1 0
0 0 0 0 0 1 1 0 0 1 1 1 0 0 0	0 0 0 0 0 1 1 0 0 1 1 1
0 0 0 0 0 1 1 0 0 0 1 1 1 0 0	1 0 0 1 0 1 0 0 0 1 1 1
0 0 0 0 0 1 1 0 0 0 0 1 1 1 0	1 1 0 1 1 1 0 1 0 1 1 1
0 0 0 0 0 1 1 0 0 0 0 0 1 1 1	1 1 1 1 1 0 0 1 1 1 1 1
0 0 0 0 0 1 0 1 0 0 0 0 0 0 0	0 0 0 0 0 1 0 1 0 0 0 0
0 0 0 0 0 1 0 1 1 0 0 0 0 0 0	0 0 0 0 0 1 0 1 1 0 0 0
0 0 0 0 0 1 0 1 0 1 0 0 0 0 0	0 0 0 0 0 1 0 1 0 1 0 0
0 0 0 0 0 1 0 1 0 0 1 0 0 0 0	0 0 0 0 0 1 0 1 0 0 1 0
0 0 0 0 0 1 0 1 0 0 0 1 0 0 0	0 0 0 0 0 1 0 1 0 0 0 1
0 0 0 0 0 1 0 1 0 0 0 0 1 0 0	1 0 0 1 0 1 1 1 0 1 0 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 0 0 1 0 1 0 0 0 0 0 1 0	0 1 0 0 1 1 0 0 0 0 1 0
0 0 0 0 0 1 0 1 0 0 0 0 0 0 1	0 0 1 0 0 0 0 1 1 0 0 1
0 0 0 0 0 1 0 1 1 1 0 0 0 0 0	0 0 0 0 0 1 0 1 1 1 0 0
0 0 0 0 0 1 0 1 0 1 1 0 0 0 0	0 0 0 0 0 1 0 1 0 1 1 0
0 0 0 0 0 1 0 1 0 0 1 1 0 0 0	0 0 0 0 0 1 0 1 0 0 1 1
0 0 0 0 0 1 0 1 0 0 0 1 1 0 0	1 0 0 1 0 1 1 1 0 1 0 1
0 0 0 0 0 1 0 1 0 0 0 0 1 1 0	1 1 0 1 1 1 1 0 0 1 1 0
0 0 0 0 0 1 0 1 0 0 0 0 0 1 1	0 1 1 0 1 0 0 0 1 0 1 1
0 0 0 0 0 1 0 1 1 0 1 0 0 0 0	0 0 0 0 0 1 0 1 1 0 1 0
0 0 0 0 0 1 0 1 0 1 0 1 0 0 0	0 0 0 0 0 1 0 1 0 1 0 1
0 0 0 0 0 1 0 1 0 0 1 0 1 0 0	1 0 0 1 0 1 1 1 0 1 1 0
0 0 0 0 0 1 0 1 0 0 0 1 0 1 0	0 1 0 0 1 1 0 0 0 0 1 1
0 0 0 0 0 1 0 1 0 0 0 0 1 0 1	1 0 1 1 0 0 1 1 1 1 0 1
0 0 0 0 0 1 0 1 1 1 1 0 0 0 0	0 0 0 0 0 1 0 1 1 1 1 0
0 0 0 0 0 1 0 1 0 1 1 1 0 0 0	0 0 0 0 0 1 0 1 0 1 1 1
0 0 0 0 0 1 0 1 0 0 1 1 1 0 0	1 0 0 1 0 1 1 1 0 1 1 1
0 0 0 0 0 1 0 1 0 0 0 1 1 1 0	1 1 0 1 1 1 1 0 0 1 1 1
0 0 0 0 0 1 0 1 0 0 0 0 1 1 1	1 1 1 1 1 0 1 0 1 1 1 1
0 0 0 0 0 1 1 1 0 0 0 0 0 0 0	0 0 0 0 0 1 1 1 0 0 0 0
0 0 0 0 0 1 1 1 1 0 0 0 0 0 0	0 0 0 0 0 1 1 1 1 0 0 0
0 0 0 0 0 1 1 1 0 1 0 0 0 0 0	0 0 0 0 0 1 1 1 0 1 0 0
0 0 0 0 0 1 1 1 0 0 1 0 0 0 0	0 0 0 0 0 1 1 1 0 0 1 0
0 0 0 0 0 1 1 1 0 0 0 1 0 0 0	0 0 0 0 0 1 1 1 0 0 0 1
0 0 0 0 0 1 1 1 0 0 0 0 1 0 0	1 0 0 1 0 1 0 1 0 1 0 0
0 0 0 0 0 1 1 1 0 0 0 0 0 1 0	0 1 0 0 1 1 1 0 0 0 1 0
0 0 0 0 0 1 1 1 0 0 0 0 0 0 1	0 0 1 0 0 0 1 1 1 0 0 1
0 0 0 0 0 1 1 1 1 1 0 0 0 0 0	0 0 0 0 0 1 1 1 1 1 0 0
0 0 0 0 0 1 1 1 0 1 1 0 0 0 0	0 0 0 0 0 1 1 1 0 1 1 0
0 0 0 0 0 1 1 1 0 0 1 1 0 0 0	0 0 0 0 0 1 1 1 0 0 1 1
0 0 0 0 0 1 1 1 0 0 0 1 1 0 0	1 0 0 1 0 1 0 1 0 1 0 1
0 0 0 0 0 1 1 1 0 0 0 0 1 1 0	1 1 0 1 1 1 0 0 0 1 1 0
0 0 0 0 0 1 1 1 0 0 0 0 0 1 1	0 1 1 0 1 0 1 0 1 0 1 1
0 0 0 0 0 1 1 1 1 0 1 0 0 0 0	0 0 0 0 0 1 1 1 1 0 1 0
0 0 0 0 0 1 1 1 0 1 0 1 0 0 0	0 0 0 0 0 1 1 1 0 1 0 1
0 0 0 0 0 1 1 1 0 0 1 0 1 0 0	1 0 0 1 0 1 0 1 0 1 1 0
0 0 0 0 0 1 1 1 0 0 0 1 0 1 0	0 1 0 0 1 1 1 0 0 0 1 1
0 0 0 0 0 1 1 1 0 0 0 0 1 0 1	1 0 1 1 0 0 0 1 1 1 0 1
0 0 0 0 0 1 1 1 1 1 1 0 0 0 0	0 0 0 0 0 1 1 1 1 1 1 0
0 0 0 0 0 1 1 1 0 1 1 1 0 0 0	0 0 0 0 0 1 1 1 0 1 1 1

Error vectors	Syndromes
0 0 0 0 0 1 1 1 0 0 1 1 1 0 0	1 0 0 1 0 1 0 1 0 1 1 1
0 0 0 0 0 1 1 1 0 0 0 1 1 1 0	1 1 0 1 1 1 0 0 0 1 1 1
0 0 0 0 0 1 1 1 0 0 0 0 1 1 1	1 1 1 1 1 0 0 0 1 1 1 1
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 1 0 0 1 0 0 0 0 0	0 0 0 0 0 0 1 0 0 1 0 0
0 0 0 0 0 0 1 0 0 0 1 0 0 0 0	0 0 0 0 0 0 1 0 0 0 1 0
0 0 0 0 0 0 1 0 0 0 0 1 0 0 0	0 0 0 0 0 0 1 0 0 0 0 1
0 0 0 0 0 0 1 0 0 0 0 0 1 0 0	1 0 0 1 0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 1 0	0 1 0 0 1 0 1 1 0 0 1 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 1	0 0 1 0 0 1 1 0 1 0 0 1
0 0 0 0 0 0 1 0 0 1 1 0 0 0 0	0 0 0 0 0 0 1 0 0 1 1 0
0 0 0 0 0 0 1 0 0 0 1 1 0 0 0	0 0 0 0 0 0 1 0 0 0 1 1
0 0 0 0 0 0 1 0 0 0 0 1 1 0 0	1 0 0 1 0 0 0 0 0 1 0 1
0 0 0 0 0 0 1 0 0 0 0 0 1 1 0	1 1 0 1 1 0 0 1 0 1 1 0
0 0 0 0 0 0 1 0 0 0 0 0 0 1 1	0 1 1 0 1 1 1 1 1 0 1 1
0 0 0 0 0 0 1 0 0 1 0 1 0 0 0	0 0 0 0 0 0 1 0 0 1 0 1
0 0 0 0 0 0 1 0 0 0 1 0 1 0 0	1 0 0 1 0 0 0 0 0 1 1 0
0 0 0 0 0 0 1 0 0 0 0 1 0 1 0	0 1 0 0 1 0 1 1 0 0 1 1
0 0 0 0 0 0 1 0 0 0 0 0 1 0 1	1 0 1 1 0 1 0 0 1 1 0 1
0 0 0 0 0 0 1 0 0 1 1 1 0 0 0	0 0 0 0 0 0 1 0 0 1 1 1
0 0 0 0 0 0 1 0 0 0 1 1 1 0 0	1 0 0 1 0 0 0 0 0 1 1 1
0 0 0 0 0 0 1 0 0 0 0 1 1 1 0	1 1 0 1 1 0 0 1 0 1 1 1
0 0 0 0 0 0 1 0 0 0 0 0 1 1 1	1 1 1 1 1 1 0 1 1 1 1 1
0 0 0 0 0 0 1 1 0 0 0 0 0 0 0	0 0 0 0 0 0 1 1 0 0 0 0
0 0 0 0 0 0 1 1 0 1 0 0 0 0 0	0 0 0 0 0 0 1 1 0 1 0 0
0 0 0 0 0 0 1 1 0 0 1 0 0 0 0	0 0 0 0 0 0 1 1 0 0 1 0
0 0 0 0 0 0 1 1 0 0 0 1 0 0 0	0 0 0 0 0 0 1 1 0 0 0 1
0 0 0 0 0 0 1 1 0 0 0 0 1 0 0	1 0 0 1 0 0 0 1 0 1 0 0
0 0 0 0 0 0 1 1 0 0 0 0 0 1 0	0 1 0 0 1 0 1 0 0 0 1 0
0 0 0 0 0 0 1 1 0 0 0 0 0 0 1	0 0 1 0 0 1 1 1 1 0 0 1
0 0 0 0 0 0 1 1 0 1 1 0 0 0 0	0 0 0 0 0 0 1 1 0 1 1 0
0 0 0 0 0 0 1 1 0 0 1 1 0 0 0	0 0 0 0 0 0 1 1 0 0 1 1
0 0 0 0 0 0 1 1 0 0 0 1 1 0 0	1 0 0 1 0 0 0 1 0 1 0 1
0 0 0 0 0 0 1 1 0 0 0 0 1 1 0	1 1 0 1 1 0 0 0 0 1 1 0
0 0 0 0 0 0 1 1 0 0 0 0 0 1 1	0 1 1 0 1 1 1 0 1 0 1 1
0 0 0 0 0 0 1 1 0 1 0 1 0 0 0	0 0 0 0 0 0 1 1 0 1 0 1
0 0 0 0 0 0 1 1 0 0 1 0 1 0 0	1 0 0 1 0 0 0 1 0 1 1 0
0 0 0 0 0 0 1 1 0 0 0 1 0 1 0	0 1 0 0 1 0 1 0 0 0 1 1
0 0 0 0 0 0 1 1 0 0 0 0 1 0 1	1 0 1 1 0 1 0 1 1 1 0 1

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 0 0 0 1 1 0 1 1 1 0 0 0	0 0 0 0 0 0 1 1 0 1 1 1
0 0 0 0 0 0 1 1 0 0 1 1 1 0 0	1 0 0 1 0 0 0 1 0 1 1 1
0 0 0 0 0 0 1 1 0 0 0 1 1 1 0	1 1 0 1 1 0 0 0 0 1 1 1
0 0 0 0 0 0 1 1 0 0 0 0 1 1 1	1 1 1 1 1 1 0 0 1 1 1 1
0 0 0 0 0 0 1 0 1 0 0 0 0 0 0	0 0 0 0 0 0 1 0 1 0 0 0
0 0 0 0 0 0 1 0 1 1 0 0 0 0 0	0 0 0 0 0 0 1 0 1 1 0 0
0 0 0 0 0 0 1 0 1 0 1 0 0 0 0	0 0 0 0 0 0 1 0 1 0 1 0
0 0 0 0 0 0 1 0 1 0 0 1 0 0 0	0 0 0 0 0 0 1 0 1 0 0 1
0 0 0 0 0 0 1 0 1 0 0 0 1 0 0	1 0 0 1 0 0 0 0 1 1 0 0
0 0 0 0 0 0 1 0 1 0 0 0 0 1 0	0 1 0 0 1 0 1 1 1 0 1 0
0 0 0 0 0 0 1 0 1 0 0 0 0 0 1	0 0 1 0 0 1 1 0 0 0 0 1
0 0 0 0 0 0 1 0 1 1 1 0 0 0 0	0 0 0 0 0 0 1 0 1 1 1 0
0 0 0 0 0 0 1 0 1 0 1 1 0 0 0	0 0 0 0 0 0 1 0 1 0 1 1
0 0 0 0 0 0 1 0 1 0 0 1 1 0 0	1 0 0 1 0 0 0 0 1 1 0 1
0 0 0 0 0 0 1 0 1 0 0 0 1 1 0	1 1 0 1 1 0 0 1 1 1 1 0
0 0 0 0 0 0 1 0 1 0 0 0 0 1 1	0 1 1 0 1 1 1 1 0 0 1 1
0 0 0 0 0 0 1 0 1 1 0 1 0 0 0	0 0 0 0 0 0 1 0 1 1 0 1
0 0 0 0 0 0 1 0 1 0 1 0 1 0 0	1 0 0 1 0 0 0 0 1 1 1 0
0 0 0 0 0 0 1 0 1 0 0 1 0 1 0	0 1 0 0 1 0 1 1 1 0 1 1
0 0 0 0 0 0 1 0 1 0 0 0 1 0 1	1 0 1 1 0 1 0 0 0 1 0 1
0 0 0 0 0 0 1 0 1 1 1 1 0 0 0	0 0 0 0 0 0 1 0 1 1 1 1
0 0 0 0 0 0 1 0 1 0 1 1 1 0 0	1 0 0 1 0 0 0 0 1 1 1 1
0 0 0 0 0 0 1 0 1 0 0 1 1 1 0	1 1 0 1 1 0 0 1 1 1 1 1
0 0 0 0 0 0 1 0 1 0 0 0 1 1 1	1 1 1 1 1 1 0 1 0 1 1 1
0 0 0 0 0 0 1 1 1 0 0 0 0 0 0	0 0 0 0 0 0 1 1 1 0 0 0
0 0 0 0 0 0 1 1 1 1 0 0 0 0 0	0 0 0 0 0 0 1 1 1 1 0 0
0 0 0 0 0 0 1 1 1 0 1 0 0 0 0	0 0 0 0 0 0 1 1 1 0 1 0
0 0 0 0 0 0 1 1 1 0 0 1 0 0 0	0 0 0 0 0 0 1 1 1 0 0 1
0 0 0 0 0 0 1 1 1 0 0 0 1 0 0	1 0 0 1 0 0 0 1 1 1 0 0
0 0 0 0 0 0 1 1 1 0 0 0 0 1 0	0 1 0 0 1 0 1 0 1 0 1 0
0 0 0 0 0 0 1 1 1 0 0 0 0 0 1	0 0 1 0 0 1 1 1 0 0 0 1
0 0 0 0 0 0 1 1 1 1 1 0 0 0 0	0 0 0 0 0 0 1 1 1 1 1 0
0 0 0 0 0 0 1 1 1 0 1 1 0 0 0	0 0 0 0 0 0 1 1 1 0 1 1
0 0 0 0 0 0 1 1 1 0 0 1 1 0 0	1 0 0 1 0 0 0 1 1 1 0 1
0 0 0 0 0 0 1 1 1 0 0 0 1 1 0	1 1 0 1 1 0 0 0 1 1 1 0
0 0 0 0 0 0 1 1 1 0 0 0 0 1 1	0 1 1 0 1 1 1 0 0 0 1 1
0 0 0 0 0 0 1 1 1 1 0 1 0 0 0	0 0 0 0 0 0 1 1 1 1 0 1
0 0 0 0 0 0 1 1 1 0 1 0 1 0 0	1 0 0 1 0 0 0 1 1 1 1 0
0 0 0 0 0 0 1 1 1 0 0 1 0 1 0	0 1 0 0 1 0 1 0 1 0 1 1

Error vectors	Syndromes
0 0 0 0 0 0 1 1 1 0 0 0 1 0 1	1 0 1 1 0 1 0 1 0 1 0 1
0 0 0 0 0 0 1 1 1 1 1 1 0 0 0	0 0 0 0 0 0 1 1 1 1 1 1
0 0 0 0 0 0 1 1 1 0 1 1 1 0 0	1 0 0 1 0 0 0 1 1 1 1 1
0 0 0 0 0 0 1 1 1 0 0 1 1 1 0	1 1 0 1 1 0 0 0 1 1 1 1
0 0 0 0 0 0 1 1 1 0 0 0 1 1 1	1 1 1 1 1 1 0 0 0 1 1 1
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0	0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 1 0 0 1 0 0 0 0	0 0 0 0 0 0 0 1 0 0 1 0
0 0 0 0 0 0 0 1 0 0 0 1 0 0 0	0 0 0 0 0 0 0 1 0 0 0 1
0 0 0 0 0 0 0 1 0 0 0 0 1 0 0	1 0 0 1 0 0 1 1 0 1 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 1 0	0 1 0 0 1 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 1	0 0 1 0 0 1 0 1 1 0 0 1
0 0 0 0 0 0 0 1 0 0 1 1 0 0 0	0 0 0 0 0 0 0 1 0 0 1 1
0 0 0 0 0 0 0 1 0 0 0 1 1 0 0	1 0 0 1 0 0 1 1 0 1 0 1
0 0 0 0 0 0 0 1 0 0 0 0 1 1 0	1 1 0 1 1 0 1 0 0 1 1 0
0 0 0 0 0 0 0 1 0 0 0 0 0 1 1	0 1 1 0 1 1 0 0 1 0 1 1
0 0 0 0 0 0 0 1 0 0 1 0 1 0 0	1 0 0 1 0 0 1 1 0 1 1 0
0 0 0 0 0 0 0 1 0 0 0 1 0 1 0	0 1 0 0 1 0 0 0 0 0 1 1
0 0 0 0 0 0 0 1 0 0 0 0 1 0 1	1 0 1 1 0 1 1 1 1 1 0 1
0 0 0 0 0 0 0 1 0 0 1 1 1 0 0	1 0 0 1 0 0 1 1 0 1 1 1
0 0 0 0 0 0 0 1 0 0 0 1 1 1 0	1 1 0 1 1 0 1 0 0 1 1 1
0 0 0 0 0 0 0 1 0 0 0 0 1 1 1	1 1 1 1 1 1 1 0 1 1 1 1
0 0 0 0 0 0 0 1 1 0 0 0 0 0 0	0 0 0 0 0 0 0 1 1 0 0 0
0 0 0 0 0 0 0 1 1 0 1 0 0 0 0	0 0 0 0 0 0 0 1 1 0 1 0
0 0 0 0 0 0 0 1 1 0 0 1 0 0 0	0 0 0 0 0 0 0 1 1 0 0 1
0 0 0 0 0 0 0 1 1 0 0 0 1 0 0	1 0 0 1 0 0 1 1 1 1 0 0
0 0 0 0 0 0 0 1 1 0 0 0 0 1 0	0 1 0 0 1 0 0 0 1 0 1 0
0 0 0 0 0 0 0 1 1 0 0 0 0 0 1	0 0 1 0 0 1 0 1 0 0 0 1
0 0 0 0 0 0 0 1 1 0 1 1 0 0 0	0 0 0 0 0 0 0 1 1 0 1 1
0 0 0 0 0 0 0 1 1 0 0 1 1 0 0	1 0 0 1 0 0 1 1 1 1 0 1
0 0 0 0 0 0 0 1 1 0 0 0 1 1 0	1 1 0 1 1 0 1 0 1 1 1 0
0 0 0 0 0 0 0 1 1 0 0 0 0 1 1	0 1 1 0 1 1 0 0 0 0 1 1
0 0 0 0 0 0 0 1 1 0 1 0 1 0 0	1 0 0 1 0 0 1 1 1 1 1 0
0 0 0 0 0 0 0 1 1 0 0 1 0 1 0	0 1 0 0 1 0 0 0 1 0 1 1
0 0 0 0 0 0 0 1 1 0 0 0 1 0 1	1 0 1 1 0 1 1 1 0 1 0 1
0 0 0 0 0 0 0 1 1 0 1 1 1 0 0	1 0 0 1 0 0 1 1 1 1 1 1
0 0 0 0 0 0 0 1 1 0 0 1 1 1 0	1 1 0 1 1 0 1 0 1 1 1 1
0 0 0 0 0 0 0 1 1 0 0 0 1 1 1	1 1 1 1 1 1 1 0 0 1 1 1
0 0 0 0 0 0 0 1 0 1 0 0 0 0 0	0 0 0 0 0 0 0 1 0 1 0 0
0 0 0 0 0 0 0 1 0 1 1 0 0 0 0	0 0 0 0 0 0 0 1 0 1 1 0

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 0 0 0 0 1 0 1 0 1 0 0 0	0 0 0 0 0 0 0 1 0 1 0 1
0 0 0 0 0 0 0 1 0 1 0 0 1 0 0	1 0 0 1 0 0 1 1 0 0 0 0
0 0 0 0 0 0 0 1 0 1 0 0 0 1 0	0 1 0 0 1 0 0 0 0 1 1 0
0 0 0 0 0 0 0 1 0 1 0 0 0 0 1	0 0 1 0 0 1 0 1 1 1 0 1
0 0 0 0 0 0 0 1 0 1 1 1 0 0 0	0 0 0 0 0 0 0 1 0 1 1 1
0 0 0 0 0 0 0 1 0 1 0 1 1 0 0	1 0 0 1 0 0 1 1 0 0 0 1
0 0 0 0 0 0 0 1 0 1 0 0 1 1 0	1 1 0 1 1 0 1 0 0 0 1 0
0 0 0 0 0 0 0 1 0 1 0 0 0 1 1	0 1 1 0 1 1 0 0 1 1 1 1
0 0 0 0 0 0 0 1 0 1 1 0 1 0 0	1 0 0 1 0 0 1 1 0 0 1 0
0 0 0 0 0 0 0 1 0 1 0 1 0 1 0	0 1 0 0 1 0 0 0 0 1 1 1
0 0 0 0 0 0 0 1 0 1 0 0 1 0 1	1 0 1 1 0 1 1 1 1 0 0 1
0 0 0 0 0 0 0 1 0 1 1 1 1 0 0	1 0 0 1 0 0 1 1 0 0 1 1
0 0 0 0 0 0 0 1 0 1 0 1 1 1 0	1 1 0 1 1 0 1 0 0 0 1 1
0 0 0 0 0 0 0 1 0 1 0 0 1 1 1	1 1 1 1 1 1 1 0 1 0 1 1
0 0 0 0 0 0 0 1 1 1 0 0 0 0 0	0 0 0 0 0 0 0 1 1 1 0 0
0 0 0 0 0 0 0 1 1 1 1 0 0 0 0	0 0 0 0 0 0 0 1 1 1 1 0
0 0 0 0 0 0 0 1 1 1 0 1 0 0 0	0 0 0 0 0 0 0 1 1 1 0 1
0 0 0 0 0 0 0 1 1 1 0 0 1 0 0	1 0 0 1 0 0 1 1 1 0 0 0
0 0 0 0 0 0 0 1 1 1 0 0 0 1 0	0 1 0 0 1 0 0 0 1 1 1 0
0 0 0 0 0 0 0 1 1 1 0 0 0 0 1	0 0 1 0 0 1 0 1 0 1 0 1
0 0 0 0 0 0 0 1 1 1 1 1 0 0 0	0 0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 0 0 1 1 1 0 1 1 0 0	1 0 0 1 0 0 1 1 1 0 0 1
0 0 0 0 0 0 0 1 1 1 0 0 1 1 0	1 1 0 1 1 0 1 0 1 0 1 0
0 0 0 0 0 0 0 1 1 1 0 0 0 1 1	0 1 1 0 1 1 0 0 0 1 1 1
0 0 0 0 0 0 0 1 1 1 1 0 1 0 0	1 0 0 1 0 0 1 1 1 0 1 0
0 0 0 0 0 0 0 1 1 1 0 1 0 1 0	0 1 0 0 1 0 0 0 1 1 1 1
0 0 0 0 0 0 0 1 1 1 0 0 1 0 1	1 0 1 1 0 1 1 1 0 0 0 1
0 0 0 0 0 0 0 1 1 1 1 1 1 0 0	1 0 0 1 0 0 1 1 1 0 1 1
0 0 0 0 0 0 0 1 1 1 0 1 1 1 0	1 1 0 1 1 0 1 0 1 0 1 1
0 0 0 0 0 0 0 1 1 1 0 0 1 1 1	1 1 1 1 1 1 1 0 0 0 1 1
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0	0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 1 0 0 1 0 0 0	0 0 0 0 0 0 0 0 1 0 0 1
0 0 0 0 0 0 0 0 1 0 0 0 1 0 0	1 0 0 1 0 0 1 0 1 1 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 1 0	0 1 0 0 1 0 0 1 1 0 1 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 1	0 0 1 0 0 1 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1 0 0 1 1 0 0	1 0 0 1 0 0 1 0 1 1 0 1
0 0 0 0 0 0 0 0 1 0 0 0 1 1 0	1 1 0 1 1 0 1 1 1 1 1 0
0 0 0 0 0 0 0 0 1 0 0 0 0 1 1	0 1 1 0 1 1 0 1 0 0 1 1
0 0 0 0 0 0 0 0 1 0 0 1 0 1 0	0 1 0 0 1 0 0 1 1 0 1 1

Error vectors	Syndromes
0 0 0 0 0 0 0 0 1 0 0 0 1 0 1	1 0 1 1 0 1 1 0 0 1 0 1
0 0 0 0 0 0 0 0 1 0 0 1 1 1 0	1 1 0 1 1 0 1 1 1 1 1 1
0 0 0 0 0 0 0 0 1 0 0 0 1 1 1	1 1 1 1 1 1 1 1 0 1 1 1
0 0 0 0 0 0 0 0 1 1 0 0 0 0 0	0 0 0 0 0 0 0 0 1 1 0 0
0 0 0 0 0 0 0 0 1 1 0 1 0 0 0	0 0 0 0 0 0 0 0 1 1 0 1
0 0 0 0 0 0 0 0 1 1 0 0 1 0 0	1 0 0 1 0 0 1 0 1 0 0 0
0 0 0 0 0 0 0 0 1 1 0 0 0 1 0	0 1 0 0 1 0 0 1 1 1 1 0
0 0 0 0 0 0 0 0 1 1 0 0 0 0 1	0 0 1 0 0 1 0 0 0 1 0 1
0 0 0 0 0 0 0 0 1 1 0 1 1 0 0	1 0 0 1 0 0 1 0 1 0 0 1
0 0 0 0 0 0 0 0 1 1 0 0 1 1 0	1 1 0 1 1 0 1 1 1 0 1 0
0 0 0 0 0 0 0 0 1 1 0 0 0 1 1	0 1 1 0 1 1 0 1 0 1 1 1
0 0 0 0 0 0 0 0 1 1 0 1 0 1 0	0 1 0 0 1 0 0 1 1 1 1 1
0 0 0 0 0 0 0 0 1 1 0 0 1 0 1	1 0 1 1 0 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0 1 1 0 1 1 1 0	1 1 0 1 1 0 1 1 1 0 1 1
0 0 0 0 0 0 0 0 1 1 0 0 1 1 1	1 1 1 1 1 1 1 1 0 0 1 1
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0	0 0 0 0 0 0 0 0 1 0 1 0
0 0 0 0 0 0 0 0 1 0 1 1 0 0 0	0 0 0 0 0 0 0 0 1 0 1 1
0 0 0 0 0 0 0 0 1 0 1 0 1 0 0	1 0 0 1 0 0 1 0 1 1 1 0
0 0 0 0 0 0 0 0 1 0 1 0 0 1 0	0 1 0 0 1 0 0 1 1 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 1	0 0 1 0 0 1 0 0 0 0 1 1
0 0 0 0 0 0 0 0 1 0 1 1 1 0 0	1 0 0 1 0 0 1 0 1 1 1 1
0 0 0 0 0 0 0 0 1 0 1 0 1 1 0	1 1 0 1 1 0 1 1 1 1 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 1 1	0 1 1 0 1 1 0 1 0 0 0 1
0 0 0 0 0 0 0 0 1 0 1 1 0 1 0	0 1 0 0 1 0 0 1 1 0 0 1
0 0 0 0 0 0 0 0 1 0 1 0 1 0 1	1 0 1 1 0 1 1 0 0 1 1 1
0 0 0 0 0 0 0 0 1 0 1 1 1 1 0	1 1 0 1 1 0 1 1 1 1 0 1
0 0 0 0 0 0 0 0 1 0 1 0 1 1 1	1 1 1 1 1 1 1 1 0 1 0 1
0 0 0 0 0 0 0 0 1 1 1 0 0 0 0	0 0 0 0 0 0 0 0 1 1 1 0
0 0 0 0 0 0 0 0 1 1 1 1 0 0 0	0 0 0 0 0 0 0 0 1 1 1 1
0 0 0 0 0 0 0 0 1 1 1 0 1 0 0	1 0 0 1 0 0 1 0 1 0 1 0
0 0 0 0 0 0 0 0 1 1 1 0 0 1 0	0 1 0 0 1 0 0 1 1 1 0 0
0 0 0 0 0 0 0 0 1 1 1 0 0 0 1	0 0 1 0 0 1 0 0 0 1 1 1
0 0 0 0 0 0 0 0 1 1 1 1 1 0 0	1 0 0 1 0 0 1 0 1 0 1 1
0 0 0 0 0 0 0 0 1 1 1 0 1 1 0	1 1 0 1 1 0 1 1 1 0 0 0
0 0 0 0 0 0 0 0 1 1 1 0 0 1 1	0 1 1 0 1 1 0 1 0 1 0 1
0 0 0 0 0 0 0 0 1 1 1 1 0 1 0	0 1 0 0 1 0 0 1 1 1 0 1
0 0 0 0 0 0 0 0 1 1 1 0 1 0 1	1 0 1 1 0 1 1 0 0 0 1 1
0 0 0 0 0 0 0 0 1 1 1 1 1 1 0	1 1 0 1 1 0 1 1 1 0 0 1
0 0 0 0 0 0 0 0 1 1 1 0 1 1 1	1 1 1 1 1 1 1 1 0 0 0 1

Table 1 (*Continued*)

Error vectors	Syndromes
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0	0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 1 0 0 1 0 0	1 0 0 1 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 1 0	0 1 0 0 1 0 0 1 0 1 1 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 1	0 0 1 0 0 1 0 0 1 1 0 1
0 0 0 0 0 0 0 0 0 1 0 0 1 1 0	1 1 0 1 1 0 1 1 0 0 1 0
0 0 0 0 0 0 0 0 0 1 0 0 0 1 1	0 1 1 0 1 1 0 1 1 1 1 1
0 0 0 0 0 0 0 0 0 1 0 0 1 0 1	1 0 1 1 0 1 1 0 1 0 0 1
0 0 0 0 0 0 0 0 0 1 0 0 1 1 1	1 1 1 1 1 1 1 1 1 0 1 1
0 0 0 0 0 0 0 0 0 1 1 0 0 0 0	0 0 0 0 0 0 0 0 0 1 1 0
0 0 0 0 0 0 0 0 0 1 1 0 1 0 0	1 0 0 1 0 0 1 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1 1 0 0 1 0	0 1 0 0 1 0 0 1 0 1 0 0
0 0 0 0 0 0 0 0 0 1 1 0 0 0 1	0 0 1 0 0 1 0 0 1 1 1 1
0 0 0 0 0 0 0 0 0 1 1 0 1 1 0	1 1 0 1 1 0 1 1 0 0 0 0
0 0 0 0 0 0 0 0 0 1 1 0 0 1 1	0 1 1 0 1 1 0 1 1 1 0 1
0 0 0 0 0 0 0 0 0 1 1 0 1 0 1	1 0 1 1 0 1 1 0 1 0 1 1
0 0 0 0 0 0 0 0 0 1 1 0 1 1 1	1 1 1 1 1 1 1 1 1 0 0 1
0 0 0 0 0 0 0 0 0 1 0 1 0 0 0	0 0 0 0 0 0 0 0 0 1 0 1
0 0 0 0 0 0 0 0 0 1 0 1 1 0 0	1 0 0 1 0 0 1 0 0 0 0 1
0 0 0 0 0 0 0 0 0 1 0 1 0 1 0	0 1 0 0 1 0 0 1 0 1 1 1
0 0 0 0 0 0 0 0 0 1 0 1 0 0 1	0 0 1 0 0 1 0 0 1 1 0 0
0 0 0 0 0 0 0 0 0 1 0 1 1 1 0	1 1 0 1 1 0 1 1 0 0 1 1
0 0 0 0 0 0 0 0 0 1 0 1 0 1 1	0 1 1 0 1 1 0 1 1 1 1 0
0 0 0 0 0 0 0 0 0 1 0 1 1 0 1	1 0 1 1 0 1 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 1 0 1 1 1 1	1 1 1 1 1 1 1 1 1 0 1 0
0 0 0 0 0 0 0 0 0 1 1 1 0 0 0	0 0 0 0 0 0 0 0 0 1 1 1
0 0 0 0 0 0 0 0 0 1 1 1 1 0 0	1 0 0 1 0 0 1 0 0 0 1 1
0 0 0 0 0 0 0 0 0 1 1 1 0 1 0	0 1 0 0 1 0 0 1 0 1 0 1
0 0 0 0 0 0 0 0 0 1 1 1 0 0 1	0 0 1 0 0 1 0 0 1 1 1 0
0 0 0 0 0 0 0 0 0 1 1 1 1 1 0	1 1 0 1 1 0 1 1 0 0 0 1
0 0 0 0 0 0 0 0 0 1 1 1 0 1 1	0 1 1 0 1 1 0 1 1 1 0 0
0 0 0 0 0 0 0 0 0 1 1 1 1 0 1	1 0 1 1 0 1 1 0 1 0 1 0
0 0 0 0 0 0 0 0 0 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 0 0
0 0 0 0 0 0 0 0 0 0 1 1 1 1 0	1 1 0 1 1 0 1 1 0 1 0 1
0 0 0 0 0 0 0 0 0 0 1 1 1 0 1	1 0 1 1 0 1 1 0 1 1 1 0
0 0 0 0 0 0 0 0 0 0 1 1 1 0 0	1 0 0 1 0 0 1 0 0 1 1 1
0 0 0 0 0 0 0 0 0 0 1 1 0 1 1	0 1 1 0 1 1 0 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 0 1 0	0 1 0 0 1 0 0 1 0 0 0 1
0 0 0 0 0 0 0 0 0 0 1 1 0 0 1	0 0 1 0 0 1 0 0 1 0 1 0

Error vectors	Syndromes
0 0 0 0 0 0 0 0 0 0 1 1 0 0 0	0 0 0 0 0 0 0 0 0 0 1 1
0 0 0 0 0 0 0 0 0 0 1 0 1 1 1	1 1 1 1 1 1 1 1 1 1 0 1
0 0 0 0 0 0 0 0 0 0 1 0 1 1 0	1 1 0 1 1 0 1 1 0 1 0 0
0 0 0 0 0 0 0 0 0 0 1 0 1 0 1	1 0 1 1 0 1 1 0 1 1 1 1
0 0 0 0 0 0 0 0 0 0 1 0 1 0 0	1 0 0 1 0 0 1 0 0 1 1 0
0 0 0 0 0 0 0 0 0 0 1 0 0 1 1	0 1 1 0 1 1 0 1 1 0 0 1
0 0 0 0 0 0 0 0 0 0 1 0 0 1 0	0 1 0 0 1 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 1	0 0 1 0 0 1 0 0 1 0 1 1
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0	0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 0
0 0 0 0 0 0 0 0 0 0 0 1 1 1 0	1 1 0 1 1 0 1 1 0 1 1 1
0 0 0 0 0 0 0 0 0 0 0 1 1 0 1	1 0 1 1 0 1 1 0 1 1 0 0
0 0 0 0 0 0 0 0 0 0 0 1 1 0 0	1 0 0 1 0 0 1 0 0 1 0 1
0 0 0 0 0 0 0 0 0 0 0 1 0 1 1	0 1 1 0 1 1 0 1 1 0 1 0
0 0 0 0 0 0 0 0 0 0 0 1 0 1 0	0 1 0 0 1 0 0 1 0 0 1 1
0 0 0 0 0 0 0 0 0 0 0 1 0 0 1	0 0 1 0 0 1 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0	0 0 0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0 0 0 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 1 1 0	1 1 0 1 1 0 1 1 0 1 1 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 1	1 0 1 1 0 1 1 0 1 1 0 1
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0	1 0 0 1 0 0 1 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 1	0 1 1 0 1 1 0 1 1 0 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0	0 1 0 0 1 0 0 1 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1	0 0 1 0 0 1 0 0 1 0 0 1

In the following example, we give a parity-check matrix for a binary code which corrects 2-repeated bursts of length b or less. The matrix has been constructed by the synthesis procedure outlined in the proof of Theorem 2.2 by taking any length b.

Example 4.2. Consider a $(5b, b)$ binary code with parity-check matrix H given as

$$\begin{array}{rcl}
 & \longleftarrow 4b \longrightarrow \quad \longleftarrow b \longrightarrow & \\
H = & \begin{bmatrix}
1 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 \\
0 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 \\
0 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 \\
0 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 1 & 0 & \cdots & \cdots & 0 & 1
\end{bmatrix}_{4b\times 5b} &
\begin{matrix} \uparrow \\ b \\ \\ \downarrow \\ \uparrow \\ b \\ \\ \downarrow \\ \uparrow \\ b \\ \\ \downarrow \\ \uparrow \\ b \\ \\ \downarrow \end{matrix}
\end{array}$$

i.e.

$$H = \begin{bmatrix} & I_b \\ I_{4b} & I_b \\ & I_b \\ & I_b \end{bmatrix}_{4b\times 5b}$$

According to the condition laid down in Theorem 2.2, a column h_j can be added to H provided that it is not a linear combination of immediately preceding $(b-1)$ columns together with any three distinct sets of b or less consecutive columns each from the remaining $j-b$ columns. So we start with $(100\ldots0)$ and keep on adding the columns in H to get a $4b \times 5b$ matrix as defined above. It can be easily verified that the condition is satisfied. Now we prove that none of the 2^{4b} non-zero tuples can be added further to H to get C_{5b+1}.

Now a 2^{4b} non-zero tuple can be considered as consisting of four sets of b consecutive components where each set is one of the 2^b tuples. Thus, a 2^{4b} tuple having b consecutive zeros in the first b or last b components cannot be added to H as it is a linear combination of three distinct sets of b or less columns of H. Also, considering the first and the last b consecutive components, there will be $2^b 2^b = 2^{2b}$ patterns of components. The condition requires that any vector having first b or last b components same as a linear combination of $C_{4b+2}, C_{4b+3}, \ldots, C_{5b}$ cannot be added to H.

Thus the vectors that could have been added to H are the ones which involve C_{4b+1}. On considering the linear combination of $C_{3b+2}, C_{3b+3}, \ldots, C_{4b}$, C_{4b+1} and $C_{4b+2}, C_{4b+3}, \ldots, C_{5b}$, we find that all the remaining 2^{4b} tuples have their last b components as linear combination of $C_{3b+2}, \ldots, C_{4b+1}, C_{4b+2}$, $\ldots, C_{5b}$ and first b components as linear combination of $C_{4b+2}, \ldots, C_{5b}$. In other words, the tuple has first b components same as a linear combination of $C_{4b+2}, \ldots, C_{5b}$ and the last b components as linear combination of $C_{3b+2}, \ldots, C_{4b+1}$, $C_{4b+2}, \ldots, C_{5b}$ which gives us a linear combination of b consecutive columns $C_{3b+2}, \ldots, C_{4b+1}$ and immediately preceding $(b-1)$ columns $C_{4b+2}, \ldots, C_{5b}$ and there are at the most $2b$ consecutive remaining components and will be a linear combination of atmost two distinct sets of b or less consecutive columns. Therefore no more columns can be added to H.

Thus the $(5b, b)$ binary code which is the null space of the matrix H as constructed above will correct all 2-repeated bursts of length b or less.

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