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CONSTRUCTION OF m-REPEATED BURST ERROR CORRECTING BINARY LINEAR CODE

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The purpose of this paper is to propose a simple method of constructing a parity-check matrix for any binary linear code capable of correcting a new kind of burst error called 'm-repeated burst error of length b or less' recently introduced by the authors. Some binary codes based on the proposed technique have been illustrated. This technique for $m = 1$ helps in resolving a long standing problem of devising a systematic algorithm for the construction of a burst error correcting code.

Keywords: Error correcting code; burst error; repeated burst error; parity-check matrix.

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1. Introduction

Burst error detecting and correcting codes is a well-known topic in the theory of error correcting codes. A burst of length b may be defined as follows (see [7, 10]):

Definition 1.1. A burst of length b is a vector whose only nonzero components are among some b consecutive components, the first and the last of which is nonzero.

An upper bound on the number of parity-check digits for a burst error correcting code was obtained by Campopiano [4] which may be stated as follows:

Result 1.2 (see [4], Theorem 4.17 [10]). There exists an (n, k) linear code that corrects any single burst of length $b < \frac{n}{2}$ or less provided that

$$q^{n-k} > q^{2(b-1)}[(q-1)(n-2b+1)+1].$$

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This bound was derived by constructing a parity-check matrix of the code by a synthesis procedure. To the best of the knowledge of the authors, there is no systematic way to construct a parity-check matrix for a burst error correcting linear code.

Corresponding to the various variants of the definition of a burst, codes have been developed for the correction of multiple bursts, bursts of bursts, random multiple bursts, low-density bursts, etc. (see [3, 9, 11, 13–15] and others).

It has been observed that in very busy communication channels, errors repeat themselves. So there is a need to develop codes which can detect and correct repeated burst errors. Recently, repeated burst error detecting and correcting codes have been studied by the authors (see [5, 6]) and by Berardi *et al.* [2]. An m-repeated burst of length b has been defined as follows (see [6]):

Definition 1.3. An m-repeated burst of length b is a vector of length n whose only nonzero components are confined to m distinct sets of b consecutive components, the first and the last component of each set being nonzero.

For example, (001020024100314030100) is a 4-repeated burst of length 3 over GF(5).

Lower and upper bounds on the number of parity-check digits required for a linear code that is capable of detecting errors which are 2-repeated burst errors have been obtained by Berardi *et al.* [2]. In the same paper, codes capable of detecting and simultaneously correcting such errors have been dealt with.

Lower bound on the number of parity-check digits required for m-repeated burst error detecting linear code and codes capable of detecting and simultaneously correcting such errors have also been dealt with by the authors [6].

Lower and upper bounds on the number of parity-check digits for codes that can correct 2-repeated bursts of length b or less have been obtained by the authors [5] and then codes capable of correcting m-repeated bursts have also been considered. Bounds on the number of check digits required for codes correcting such errors have been obtained. Generalizing Result 1.2 for the correction of m-repeated burst errors as obtained in Theorem 4 (see [5]) is as follows:

Result 1.4. There shall always exist an (n, k) linear code over $GF(q)$ that corrects all m-repeated bursts of length b or less $(n > 2mb)$ provided that

$$q^{n-k} > q^{2m(b-1)}\left(\binom{n-2mb+(2m-1)}{2m-1}(q-1)^{2m-1} + \sum_{l=0}^{2m-2}\binom{n-2mb+l}{l}(q-1)^{l}q^{2m-2-l}\right).$$

The existence of such a code has again been proved as in Result 1.2 by constructing a parity-check matrix for m-repeated burst error correcting code using the technique to establish Varsharmov–Gilbert–Sacks bound. In the same paper, the authors have also considered an illustration of a $(5b, b)$ binary code correcting 2-repeated bursts of length b or less with parity-check matrix

$$H = \begin{bmatrix} & I_b \\ I_{4b} & I_b \\ & I_b \\ & I_b \end{bmatrix}. \tag{1.1}$$

The study of these codes is important not only from mathematical point of view as a generalization of burst but also because of the occurrence of such errors in other subject areas. In a recent study by Srinivas *et al.* [12], the changes in the neuronal network properties during epileptiform activity *in vitro* in planar two-dimensional neuronal networks cultured on a multielectrode array, using the *in vitro* model of stroke-induced epilepsy have been explored. Neuronal networks in culture show spontaneous firing activity with short phases of synchronized firing known as network bursts. A network burst represents the period of synchronized activity in the network. For the detection of network burst, a threshold value was calculated as product of number of spikes per time bin and number of active channels. An active channel was defined as a channel presenting two bursts in an acquisition time of 300 s. In other words, an active channel presents 2-repeated burst in an acquisition time of 300 s. Further, experiments performed on hippocampal networks showed synchronized network bursts of similar duration. Glutamate-injured networks also showed network bursts of similar durations, but the bursts occurred more frequently. In other words, the value of m for m-repeated bursts is much higher in glutamate-injured networks as compared to control networks.

The purpose of this paper is to present a very easy and simple method for the construction of a parity-check matrix of m-repeated burst error correcting code in the binary case. In what follows, a linear code will be considered as a subspace of the space of all n-tuples over $GF(q)$. The distance between two vectors shall be considered in the Hamming's sense.

2. Illustrations of Repeated Burst Error Correcting Linear Codes

In this section we provide an example of a binary linear code that can correct m-repeated burst errors of any length followed by illustrations with a specified value of m and length of burst.

Example 2.1. Consider the following binary matrix H:

$$H = \begin{bmatrix} & I_b \\ & I_b \\ & \cdot \\ I_{2mb} & \cdot \\ & \cdot \\ & I_b \\ & I_b \end{bmatrix}. \tag{2.1}$$

Such a matrix considered as a parity-check matrix shall give rise to a $((2m+1)b, b)$ binary linear code. Such a code corrects m-repeated bursts of length b or less.

Justification. According to the condition laid down in Result 1.4 (see Theorem 4 [5]), a column h_j can be added to H provided that it is not a linear combination of immediately preceding $b-1$ columns together with any $(2m-1)b$ or less consecutive columns from the remaining first $j-b$ columns. So we start with (100...0) and keep on adding the columns in H to get a $2mb \times (2m+1)b$ matrix as defined above. It can be easily verified that the condition is satisfied. Now we prove that none of the $2^{2mb}-1$ nonzero binary tuples can be added further to H to get $((2m+1)b+1)$th column.

Now a 2^{2mb} nonzero tuple can be considered as consisting of $2m$ sets of b consecutive components where each set is a 2^b-tuple. Thus, a 2^{2mb}-tuple having b consecutive zeros in the first b or last b components cannot be added to H as it is a linear combination of $(2m-1)b$ or less consecutive columns of H. Considering the first b and the last b consecutive components, there will be $2^b 2^b = 2^{2b}$ patterns. Further, the condition to add the next column requires that any vector having either first b or last b components becoming same as a linear combination of $h_{2mb+2}, h_{2mb+3}, \ldots, h_{(2m+1)b}$ cannot be added to H. Thus the linear combinations that can be added to H should be the ones which involve h_{2mb+1}. On considering the linear combination of $h_{(2m-1)b+2}, h_{(2m-1)b+3}, \ldots, h_{2mb}, h_{2mb+1}$ and $h_{2mb+2}, h_{2mb+3}, \ldots, h_{(2m+1)b}$, we find that all the remaining 2^{2b} tuples have their last b components as linear combination of $h_{(2m-1)b+2}, \ldots, h_{2mb+1}, h_{2mb+2}, \ldots, h_{(2m+1)b}$ and first b components as linear combination of $h_{2mb+2}, \ldots, h_{(2m+1)b}$. Obviously there are atmost $(2m-2)b$ consecutive remaining components and will be a linear combination of atmost $(2m-2)$ distinct sets of b or less consecutive columns. Therefore no more columns can be added to H. Thus the $((2m+1)b, b)$ binary code which is the null space of the matrix H as constructed above will correct all m-repeated bursts of length or less.

Remark. The above example shows that there is no need to construct the parity-check matrix by the synthesis procedure as laid down in the proof of Theorem 4 (see [5]). The cumbersome synthesis procedure involving unwieldy computations required in Theorem 4 (see [5]) to construct a parity-check matrix for the requisite code can be replaced by considering a matrix of the type given above saving all the computational task. It is clear that for any given feasible integer values of the parameters m and b, a matrix of the type H as in (2.1) can always be constructed and such a matrix meets the requirements of the theorem. It may be noted that this matrix can be used to correct m-repeated bursts of length b or less.

Example 2.2. For $m = 2$ and $b = 3$, the parity-check matrix for such a code is

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

It has been verified that this matrix corrects 2-repeated bursts of length 3 or less (see [5]).

Example 2.3. For $m = 1$ and $b = 3$, consider a $(9, 3)$ binary code with parity-check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Table 1 shows that that the syndromes of different bursts of length 3 or less are distinct, showing thereby that the code that is the null space of the matrix corrects all bursts of length 3 or less.

Example 2.4. For $m = 1$, consider a $(3b, b)$ binary code with parity-check matrix

$$H = \begin{bmatrix} & I_b \\ I_{2b} & \\ & I_b \end{bmatrix}. \tag{2.2}$$

As in Example 2.1 for m-repeated burst case, it can be easily verified that the above matrix as parity-check matrix gives rise to usual burst error correcting binary linear code. Such a construction of the matrix solves the long standing problem for

Table 1. Error vector-syndrome.

Error vectors	Syndromes
1 0 0 0 0 0 0 0 0	1 0 0 0 0 0
0 1 0 0 0 0 0 0 0	0 1 0 0 0 0
0 0 1 0 0 0 0 0 0	0 0 1 0 0 0
0 0 0 1 0 0 0 0 0	0 0 0 1 0 0
0 0 0 0 1 0 0 0 0	0 0 0 0 1 0
0 0 0 0 0 1 0 0 0	0 0 0 0 0 1
0 0 0 0 0 0 1 0 0	1 0 0 1 0 0
0 0 0 0 0 0 0 1 0	0 1 0 0 1 0
0 0 0 0 0 0 0 0 1	0 0 1 0 0 1
1 1 0 0 0 0 0 0 0	1 1 0 0 0 0
0 1 1 0 0 0 0 0 0	0 1 1 0 0 0
0 0 1 1 0 0 0 0 0	0 0 1 1 0 0
0 0 0 1 1 0 0 0 0	0 0 0 1 1 0
0 0 0 0 1 1 0 0 0	0 0 0 0 1 1
0 0 0 0 0 1 1 0 0	1 0 0 1 0 1
0 0 0 0 0 0 1 1 0	1 1 0 1 1 0
0 0 0 0 0 0 0 1 1	0 1 1 0 1 1
1 0 1 0 0 0 0 0 0	1 0 1 0 0 0
0 1 0 1 0 0 0 0 0	0 1 0 1 0 0
0 0 1 0 1 0 0 0 0	0 0 1 0 1 0
0 0 0 1 0 1 0 0 0	0 0 0 1 0 1
0 0 0 0 1 0 1 0 0	1 0 0 1 1 0
0 0 0 0 0 1 0 1 0	0 1 0 0 1 1
0 0 0 0 0 0 1 0 1	1 0 1 1 0 1
1 1 1 0 0 0 0 0 0	1 1 1 0 0 0
0 1 1 1 0 0 0 0 0	0 1 1 1 0 0
0 0 1 1 1 0 0 0 0	0 0 1 1 1 0
0 0 0 1 1 1 0 0 0	0 0 0 1 1 1
0 0 0 0 1 1 1 0 0	1 0 0 1 1 1
0 0 0 0 0 1 1 1 0	1 1 0 1 1 1
0 0 0 0 0 0 1 1 1	1 1 1 1 1 1

the direct and systematic method for the construction of codes to correct bursts of length b or less. As was pointed out earlier in Remark, there is no need to construct parity-check matrix by the laid down synthesis procedure as in [4] and Theorem 4.17 [10]. Such a matrix can always be constructed once the value of b is known.

Open Problem. It is not known if a matrix of type H as in (2.1) and (2.2) can be obtained for nonbinary codes.

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